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FINAL REPORT

OPERATIONS RESEARCH STUDY OF SYSTEMS
TRADE-OFFS IN A CONSTRAINED ENVIRONMENT

THE STRATEGIC BASING OF FUTURE LIMITED WAR FORCES

Contract No. Nonr 3983 (00)

Revision
29 November 1965

8696-6005-TU-R01

TRW Systems
One Space Park
Redondo Beach • California

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This is the final report of Contract Nonr-3983(00). An earlier report, dated October 31, 1963, developed the methodology utilized herein in the context of a tactical amphibious assault operation. Here the methods have been applied to the strategic basing of future limited war forces. Views and conclusions cited in this report result from the interpretations of the study analyses by the principal investigator Paul D. Chaiken at the time of issue, and do not necessarily reflect the official opinion of the Office of Naval Research or the Navy Department.

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I. SUMMARY AND CONCLUSIONS

To project men and supporting equipment from a staging base which at best could be an available friendly foreign base, or at worst must be the territorial United States, to all places of interest to the United States requires a strategic basing concept which is compatible with the tactical force's terminal objectives. This is due to the fact that the terminal activity of a tactical force will in general be constrained by the logistic characteristics of the strategic basing concept used.

An important characteristic of any strategic basing configuration is its response time to a terminal mission objective; that is, its ability to project a tactical force to the theater of operations and engage the enemy successfully. There is an implicit trade-off in this situation between the logistic characteristics of the basing configuration defined by response time and the tactical force required to achieve success. Usually, the faster the tactical force is projected, the less tactical force in terms of men and material is needed to successfully contain the situation. However, the quick strategic response time can only come with high-cost strategic systems. On the other hand, slower response time while cheaper, requires more tactical force systems because the enemy action has come closer to becoming a 'fait accompli.' The ideal situation would be to be able to project just enough limited war forces at sufficient speed to deter the opposition from acting at all.

Another trade-off influencing strategic reaction time is the disposition of deployed limited war forces throughout the world and its effect upon the terminal mission objective. It is true that the closer the advance base is to the enemy thrust, the shorter the strategic reaction time to meet the thrust from this advanced base. However, the number of limited war forces stationed in a deployed state at this base is also important. To quickly deploy into combat insufficient forces from an advanced base might result in not accomplishing the desired objectives (e.g., a successful containment of the enemy) at all. In this case, existing limited war forces would not only be wasted but a successful campaign could be further delayed in time because the situation would require the use of forces in excess of a well planned operation in order to achieve the desired results. Thus, we see that we are dealing with a problem characterized by geographical, spatial and temporal parameters, limited war force sizes and their corresponding deployment response times, terminal mission force objectives vis-a-vis the enemy threat, and logistic volumetric weight and velocity constraints. The purpose of this study then, was to synthesize these parameters into a working mathematical model which hopefully would demonstrate not only the many possible trade-offs available to the analyst, but a proper measure of merit in which to judge the myriad of scenario possibilities inherent in the strategic basing problem for limited war forces.

The model used was the same as the model developed in Volume I of this study (see Appendix A or Reference 1). This was made possible by the fact that the deployment of limited war forces whether tactically on the battlefield or strategically on or above the surface of the earth, have very similar mathematical structural characteristics. Therefore, with a reinterpretation of the Volume I model input variables, it was possible to analyze the strategic problem in basing limited war forces in a manner general enough to yield significant results.

Three scenarios were sequentially considered, each characterized with increasing complexity. The first scenario was a simple response to an aggression of a given terminal access area (TA). The response to this aggression was in the form of an attack originating from the zone of the interior (ZI) and/or an attack from prepositioned forces stationed at an advance base (AB) near the TA. Since the responder did not necessarily know which of the many possible TA's would be invaded, a penalty was imposed upon the responder if he decided to utilize AB's for prepositioning of his force for quick response. This penalty reduced the number of prepositioned forces at an AB in proportion to the increasing number of AB's required. No such penalty was imposed if the responder decided to utilize a ZI basing policy with much slower response. The aggressor was allowed a tactical choice of attacking en masse and/or at a piecemeal daily rate of flow with a fixed number of forces. The results of this analysis indicate that for a single, terminal access area conflict, the advantage gained by having "quick" response-time is lost as the amount of resources (limited war forces) which can be deployed becomes smaller. Therefore, the stationing of forces overseas, if the responder must spread his forces over many theaters of operations, is not justified. Of course, it can also be assumed that given enough time all the forces available to the responder at advanced bases could be recommitted into the conflict area, thus removing the disadvantage of precommitting forces to advance bases. This was the second scenario abstracted for analysis. The results of this recommitment scenario indicated that significant decision levels and corresponding thresholds exist (see Reference 1) which can be functionally related to the factors of distance, time and forces which characterize the scenario. Because of this, it was felt that the analysis of a realistic scenario which would define specific distance/time characteristics would be pertinent. This scenario represented the third and final analysis of this study and was entitled: "A War In Southeast Asia."

This scenario is derived from a significant and real world situation which was used as the basis for the analysis of the force structure, firepower and logistic characteristics of the strategic basing problem for limited war forces of both the United States of America and the Chinese mainland. Three general terminal access areas (TA's) were defined. They were: (1) India/Burma; (2) Korea; and (3) the general area defined by Thailand/Laos/Cambodia/South Vietnam. The United States can counter such a threat in two ways. The first by resisting the invasion of the above defined terminal access areas (TA's) by the projection of troops from the zone of the interior (ZI) and/or an advanced base such as Hawaii, Formosa, Okinawa, Philippines, Japan, Korea and South Vietnam. These latter two advance bases (AB's) are also TA's. The second method of resisting the Chinese mainland's peripheral expansion would be to directly attack China's heartland and force her to recommit her forces back to the homeland in its defense. This latter decision might force China to retain some of its forces to defend the homeland directly, rather than recommit some or all of its forces after the invasion of the TA's. Thus we have a two-sided game with the above strategic choices available to each side.

Based upon the above scenario the following question was asked:

> Is it possible for the United States, the defender, to present a force posture (i.e., in terms of number of men, firepower, response time, and logistic support) such that China, the attacker, would <u>never</u> commit all of its forces to the invasion of the TA's for optimal play of the game, if China was forced (or constrained) to commit only part of its forces to the invasion?

If the United States could present such a force posture, it could be said that the United States <u>effectively deterred</u> China, the attacker, provided China was acting in a rational manner (as defined by her optimal play of the game). For if China <u>intended</u> to attack the TA's (this intention being defined in mathematical model language as a constraint which in effect forces China to commit some of its forces against the TA's), and then examined its optimal strategy to commit its remaining forces and found that such a strategy was instead a defense of the homeland, one can question China's feeling of superiority while committing an aggressive act. In other words, China's aggression, whether intentional or not, and if properly deterred as defined above, would be stabilized in that her remaining forces would not be committed if the play of the game was to remain optimal (rational). If China was irrational, then the United States would have to present a force posture to China much in excess of deterrence such that China's degree of irrationality could be effectively lowered. This excess is not necessarily more deterrence, (which is possible because deterrence has been quantized in this study) but rather, could be more superiority (also quantized in this study). Deterrence, as defined above, is a weaker criteria than the achieving of a winning campaign. In fact one can achieve deterrence and still lose the campaign if deterrence fails because the threat is irrational. Obviously, force postures which result in deterrence but not the winning of a campaign (if fought) can be dangerous in the real world. On the other hand deterrence and the successful conclusion of the campaign can be achieved even if the aggressor acted irrationally.

The results of the analyses performed in this study can now be tabulated as follows:

> The defender (or responder to aggression) must present a credible force posture for deployment against the aggressor.

By this statement it is meant that the U. S. must be able to deploy sufficient forces and effective firepower to the TA's defined in the scenario to allow the aggressor the analytic capability of measuring its military effectiveness of continuing a conflagration once this conflagration has been started. If the U. S. commits an insufficient number of forces (plus effective firepower) no matter how fast into a TA, or can only deploy an insufficient number of forces from nearby AB's, such forces will not only tend to be wasted, but the insufficient deployment might cause the aggressor to escalate the conflict by committing even more of its forces into the conflagration on the grounds that to do so would represent an optimum allocation of its total forces.

> The defender does not deter the aggressor by presenting a force posture against the aggressor of parity. The defender needs a superior force posture for deterrence.

By parity it is meant that the defender has sufficient forces and effective firepower in a deployed state to result in a stand-off against a potential aggressor. The above result is true only if the effective firepower deployed against the aggressor by the defender is deployed uniformally against all the possible invasion areas. If on the other hand the defender deploys its effective incremental firepower against the <u>aggressor's</u> <u>homeland</u> <u>only</u>, then it is possible to deter the aggressor without having sufficient forces in a deployed state to be victorious. This is the so-called "Dulles Deterrent Philosophy" of the 1950's.

> An overseas advance basing policy for the precommitment of forces seems to have merit only when the defender's total deployable force posture is inadequate to deter the aggressor and provide victory.

Once superiority and deterrence are achieved by the defender in terms of deployable forces to meet the threat, there seems to be no functional relationship between advance basing characteristics available to the defender and the pay-offs defined by the Southeast Asia Scenario. In fact once deterrence has been achieved, a zone of the interior and/or single advanced base basing concept is the proper one.

> The decreasing of the defender's response time to the aggressor's invasion, if the defender has sufficient deployable forces to achieve deterrence, can degrade this established deterrence.

This surprising res. . is revealed when one examines the decision surface for both sides generated by the mathematical model describing the Southeast Asia Scenario where the defender's response time to the aggressor's invasion is allowed to go to zero. The abstract model interprets this situation in terms of the stability of the decision surface which in effect states that as the defender's response time is lowered (for the defender's deterrent force posture), the aggressor becomes more "trigger happy" and is less likely to maintain split forces if for some reason he was constrained to split them at the outset (forced and/o . .tended to invade terminal access areas with part of his forces).

Summarizing the above, one can state:

> The problem of strategic basing must be tied to the <u>total U.S. forces</u> projected against the aggressor and not just the limited order of battle projected by the Navy/Marine Corps.
>
> The interface problems associated with combining the Marine Corps with the Army in projecting a total U.S. Force posture overseas seems as pertinent as the wedding of the Army with the Air Force utilizing the USSTRICOM concept.
>
> There is more consistency in strategy, when attempting to relate an advance basing policy/limited war objectives, to project total U.S. forces from the zone of the interior than to precommit some of the U.S. forces to many overseas advanced bases.

> In some circumstances, it is not only sufficient to use conventional modes of transportation (e.g., surface ships) to project sufficient total U.S. forces in defense of our interests overseas, but also desirable.
>
> Control of the seas is mandatory!
>
> This analysis indicated that the USSTRICOM concept of deploying forces to meet a limited war aggression overseas has several questionable areas regarding its operational effectiveness. This concept not only reduces the effective military force that can be projected due to the weight limitations characteristic of air transportation and is extremely expensive as compared to conventional modes of transportation, it also degrades deterrence.*
>
> The Indian Ocean Task Force for the purpose of projecting limited war forces in the defense of India appears to be of questionable value if the mission of such a task force is to protect India from a Chinese threat. The number of troops and firepower required to deter China cannot possibly be stationed aboard such a task force.
>
> A nonmilitary solution to the limited war/counter insurgency problem might be provided for the long range time period. Studies should be made to determine the reasons for the U.S. to be committed to overseas nationalities and whether more flexible international policies can be made possible, especially those alternatives which do not depend only upon military solutions typified by the Southeast Asian Scenario analyzed in this study.

The last conclusion suggests to this writer that if a change in our foreign policy were possible, it could come about by re-examining this country's economic requirements as it necessitates relations with governments beyond the zone of the interior. Since the United States depends upon imports for part or most of its industrial raw materials, a significant factor in supporting the present "interdependence policy" with the free and not so free world, could be the maintaining of the availability of these needed raw materials. An alternative solution could be for this country to replace as many of these raw materials as possible by

---

*The definition of deterrence is defined in section II, E, 2, c of this study. The above USSTRICOM concept (when compared to normal modes of limited war force deployment) actually reduces the decision threshold (the measure of deterrence) of the aggressor in committing all his forces to the aggression if for any reason he was forced or intended to commit a part of his forces.

developing other sources for the materials. The oceans of the world are probably a potential source of many of the materials we need. Therefore, a systematic exploration of these large bodies of water could be the answer to this country's foreign policy dilemma, such policy, although successful in Europe, appears to be inadequate for Asia. For if the United States could replace any or all of its raw material requirements from oceanic sources, we could begin to use economic rather than military pressure to achieve our desired foreign policy objectives. Also we could blunt "once-and-for-all" the foreign policy objective of a Sino-Soviet Bloc, that is, to isolate the United States both politically and economically from the rest of the world.

II. THE STRATEGIC BASING OF LIMITED WAR FORCES

A. INTRODUCTION

To project men and supporting equipment from a staging base which at best could be an available friendly foreign base, or at worst must be the territorial United States, to all places of interest to the United States requires a strategic basing concept which is compatible with the tactical force's terminal objectives. This is due to the fact that the terminal activity of a tactical force will in general be constrained by the logistic characteristics of the strategic basing concept used.

An important characteristic of any strategic basing configuration is its response time to a terminal mission objective; that is, its ability to project a tactical force to the theater of operations and engage the enemy successfully. There is an implicit trade-off in this situation between the logistic characteristics of the basing configuration defined by response time and the tactical force required to achieve success. Usually the faster the tactical force is projected, the less tactical force in terms of men and material is needed to successfully contain the situation. However, the quick strategic response time can only come with high cost strategic systems. On the other hand, slower response time while cheaper, requires more tactical force systems because the enemy action has come closer to becoming a 'fait accompli.' The ideal situation would be to be able to project just enough limited war forces at sufficient speed to deter the opposition from acting at all.

Another trade-off influencing strategic reaction time is the disposition of deployed limited war forces throughout the world and its effect upon the terminal mission objective. It is true that the closer the advance base is to the enemy thrust, the quicker the strategic reaction time to meet the thrust from this advanced base However, the number of limited war forces stationed in a deployed state at this base is also important. To quickly deploy into combat insufficient forces from an advanced base might result in not accomplishing the desired objectives (e.g., a successful containment of the enemy) at all. In this case, existing limited war forces would not only be wasted but a successful campaign could be further delayed in time because the situation would require the use of forces in excess of a well planned operation in order to achieve the desired results. Thus, we see that we are dealing with a problem characterized by geographical, spatial and temporal parameters, limited war force sizes and their corresponding deployment response times, terminal mission force objectives vis-a-vis the enemy threat, and logistic volumetric weight and velocity constraints. The purpose of this study then, is to synthesize these parameters into a working mathematical model which hopefully would demonstrate not only the many possible trade-offs available to the analyst, but a proper measure of merit in which to judge the myriad of scenario possibilities inherent in the strategic basing problem for limited war forces.

B. STRATEGIC BASING CONFIGURATIONS

The strategic basing configurations available to the military planner for future limited war forces can be abstracted as illustrated in Figure 1. From the zone of the interior, (ZI), limited war forces can be deployed into the theater of operations via surface, subsurface, and/or air transport, either directly into combat or prepositioned to an advanced base and then to the terminal access or the conflict area. The mission objective of such a deployment of limited war forces could be simply stated as

to gain access to
to control
to possess

a given geographic area of interest to the United States. Figure 1 defines four different spatial characteristics pertinent to the strategic basing of limited war forces. The <u>zone of the interior (ZI)</u> refers to the many bases located in the continental United States whose responsibilities are to initiate and organize limited war forces for disposition throughout the world. These operational forces can be located either in the ZI or at some advanced base in a <u>theater of operations</u>. Some of these forces can be assigned to the fleet operating as a moving advanced base overseas. The <u>logistics</u> connecting the ZI, theater of operations, and <u>terminal access</u> consists of air, sea, and/or underwater transportation and is directly responsible for the strategic deployment of limited war forces (troops and supplies) throughout the world. The terminal access represents the conflict area. It should be noted that the degree of success in accomplishing the

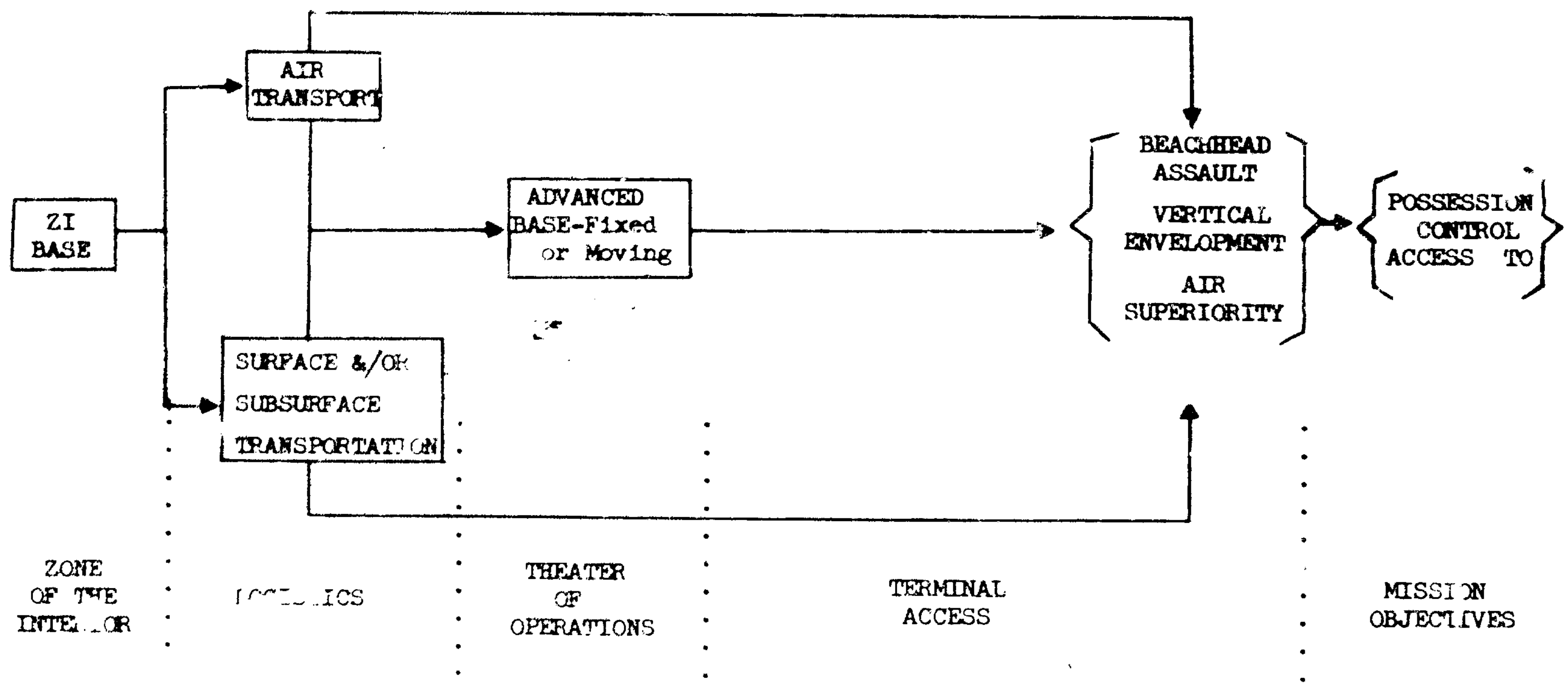

Figure 1. STRATEGIC BASING CONFIGURATIONS FOR LIMITED WAR SYSTEMS

mission objectives assigned to the limited war forces when deployed to this conflict area will in general represent the measure of merit for the complete operation (both logistic and military). In what manner should limited war forces be strategically deployed throughout the world, how fast and in what quantity should these forces be projected into the conflict area for any given enemy thrust or thrusts will be the proper concern of this study. The techniques to be employed will be very similar to those outlined in the previous volume.* That is, the mathematical structure of the above formulated problem will be of more interest to this study than numerical evaluations of specific limited war scenarios. The extent to which this objective can be accomplished will depend upon the success to which a mathematical model can be developed to fully represent the essence of the above 'strategic basing' problem.

C. THE MATHEMATICAL MODEL

The development of a mathematical model to analyze problems associated with the strategic basing of limited war forces is suggested both from Figure 1 and the results of Volume I, the volume dealing with the assault phase of the amphibious/vertical envelopment operation. Since the measure of merit to be used must reflect the outcome of the assault operation as it was functionally related to the strategic basing concept employed, some form of Lanchester's Equation might be appropriate. The attacking force y (as opposed to the defending limited war force x) initiate the action by an invasion of the terminal access area. The y forces invade this area at a given rate, $r_y(t) > 0$. The defending force x responds to this attack at different response times t_{ZI} and/or t_{AB}, depending upon x's initial disposition of limited war forces either in the zone of the interior or at an advanced base. If we define the normalized rates of attrition for x and y as a and b respectively, a simplified model of the strategic basing configuration (assuming the spatial characteristics of the operation can be approximated by one zone of the interior (ZI) base, one advance

*See Appendix A, page 62, for a brief description of the technique or the material in Reference (1) for a fuller description.

base (AB) base, and only one terminal access area) can be defined as follows:

$$\dot{y} = r_y - \Delta(t - t_{ZI})\, a_{ZI}\, x_{ZI} - \Delta(t - t_{AB})\, a_{AB}\, x_{AB}$$

$$\dot{x}_{ZI} = - \Delta(t - t_{ZI})\, b\, y$$

$$\dot{x}_{AB} = - \Delta(t - t_{AB})\, b\, y \qquad (1)$$

where $x(t=0) = x_{ZI}(0) + x_{AB}(0)$

$$y(t) = \int_0^t r_y(\xi) d\xi \text{ minus attrition losses}$$

$0 \leq t \leq T$, where T is y's invasion cutoff time

$$\Delta(t - \bar{t}) = \begin{matrix} 1 & t \geq \bar{t} \\ 0 & t < \bar{t} \end{matrix} \; ; \; \bar{t} = t_{ZI},\, t_{AB}.$$

The easiest way to solve these equations is to subdivide the time domain into three zones such that equations (1) yield continuous solutions in the time domain chosen. The pertinent time domain yielding continuous solutions is

$$0 \leq t < t_{AB}$$

$$t_{AB} \leq t < t_{ZI}$$

$$t_{ZI} \leq t. \qquad (2)$$

The above time domains assume $t_{AB} < t_{ZI}$. If for some reason $t_{ZI} < t_{AB}$, then the indices in equations (2) should be inverted.

The solutions now follow:

<u>Case I: $0 \leq t \leq t_{AB}$</u>

$$\dot{y} = r_y \qquad (3)$$

$$\left.\begin{aligned} y(t) &= \int_0^t r_y(\xi)\, d\xi \\ x_{AB}(t) &= x_{AB_0} \\ x_{ZI}(t) &= x_{ZI_0} \end{aligned}\right\} \quad 0 \leq t < t_{AB} \qquad (4)$$

Case II: $t_{AB} \le t < t_{ZI}$

$$\dot{y} = r_y - a_{AB}\, x_{AB}$$
$$\dot{x} = -\, b\, y \qquad (5)$$

where

$$y(t_{AB}) = \int_0^{t_{AB}} r_y\,(\xi)\, d\xi$$

$$x_{AB}(t_{AB}) = x_{AB_0}$$

$$x_{ZI}(t_{AB}) = x_{ZI_0}.$$

Solving equations (5) using Laplace Transforms, we have:

$$s\,\tilde{y} - y(t_{AB}) = r_{\tilde{y}} - a_{AB}\tilde{x}_{AB}$$
$$s\,\tilde{x}_{AB} - x_{AB_0} = -\, b\,\tilde{y}. \qquad (6)$$

Rewriting equations (6) and solving algebraically, we get:

$$\begin{pmatrix} s & a_{AB} \\ b & s \end{pmatrix} \begin{pmatrix} \tilde{y} \\ \tilde{x}_{AB} \end{pmatrix} = \begin{pmatrix} r_{\tilde{y}} + y(t_{AB}) \\ x_{AB_0} \end{pmatrix}. \qquad (7)$$

Let $\delta_2 = s^2 - a_{AB}\, b.$ (8)

Then

$$\tilde{y} = \frac{1}{\delta_2}\left\{\left[r_{\tilde{y}} + y(t_{AB})\right] s - a_{AB}\, x_{AB_0}\right\}$$
$$\tilde{x}_{AB} = \frac{1}{\delta_2}\left\{x_{AB_0}\, s - \left[r_{\tilde{y}} + y(t_{AB})\right] b\right\}. \qquad (9)$$

The pertinent Laplace Transforms are

$$\sinh at \longleftrightarrow \frac{a}{s^2 - a^2}$$

$$\cosh at \longleftrightarrow \frac{s}{s^2 - a^2}. \qquad (10)$$

The solution to equations (9) in the time domain $t_{AB} \leq t < t_{ZI}$ can be written:

$$y(t) = y(t_{AB}) \cosh \sqrt{a_{AB} b}\, t - \sqrt{\frac{a_{AB}}{b}}\; x_{AB_0} \sinh \sqrt{a_{AB} b}\; t + L^{-1}\left\{\frac{r_{\tilde{y}}\, s}{\delta_2}\right\}$$

$$x_{AB}(t) = x_{AB_0} \cosh \sqrt{a_{AB} b}\, t - \sqrt{\frac{b}{a_{AB}}}\; y(t_{AB}) \sinh \sqrt{a_{AB} b}\; t - L^{-1}\left\{\frac{b\, r_{\tilde{y}}}{\delta_2}\right\};\ t_{AB} \leq t < t_{ZI} \tag{11}$$

The last term in both expressions of equations (11), $L^{-1}\{.\}$, denotes the inverse Laplace Transformation of a function of $r_{\tilde{y}} \longleftrightarrow r_y(t)$ which represents y's thrust into the terminal access or combat area. An example of this type of time function would be a step or impluse function, a constant, an s-shaped curve, etc.

Case III: $t_{ZI} \leq t$

$$\begin{aligned} \dot{y} &= r_y - a_{AB}\, x_{AB} - a_{ZI}\, x_{ZI} \\ \dot{x}_{AB} &= -\, b\alpha y \qquad \text{where } (\alpha + \gamma) = 1.0 \\ \dot{x}_{ZI} &= -\, b\gamma y \end{aligned} \tag{12}$$

where

$$y(t_{ZI}) = y(t_{AB}) \cosh \sqrt{a_{AB} b}\; t_{ZI} - \sqrt{\frac{a_{AB}}{b}}\; x_{AB_0} \sinh \sqrt{a_{AB} b}\; t_{ZI} + L^{-1}\left\{\frac{r_{\tilde{y}}\, s}{\delta_2}\right\}_{t = t_{ZI}}$$

$$x_{AB}(t_{ZI}) = x_{AB_0} \cosh \sqrt{a_{AB} b}\; t_{ZI} - \sqrt{\frac{b}{a_{AB}}}\; y(t_{AB}) \sinh \sqrt{a_{AB} b}\; t_{ZI} - L^{-1}\left\{\frac{b\, r_{\tilde{y}}}{\delta_2}\right\}_{t = t_{ZI}}$$

$$x_{ZI}(t_{ZI}) = x_{ZI_0}.$$

Again using Laplace Transforms we convert equations (12) into a set of algebraic equations:

$$
\begin{aligned}
s\,\tilde{y} - y(t_{ZI}) &= \tilde{r}_y - a_{AB}\,\tilde{x}_{AB} - a_{ZI}\,\tilde{x}_{ZI} \\
s\,\tilde{x}_{AB} - x_{AB}(t_{ZI}) &= -\,b\,\alpha\,\tilde{y} \\
s\,\tilde{x}_{ZI} - x_{ZI}(t_{ZI}) &= -\,b\,\gamma\,\tilde{y}
\end{aligned}
\tag{13}
$$

Rewriting in matrix form

$$
\begin{pmatrix} s & a_{AB} & a_{ZI} \\ \alpha b & s & 0 \\ \gamma b & 0 & s \end{pmatrix}
\begin{pmatrix} \tilde{y} \\ \tilde{x}_{AB} \\ \tilde{x}_{ZI} \end{pmatrix}
=
\begin{pmatrix} \tilde{r}_y + y(t_{ZI}) \\ x_{AB}(t_{ZI}) \\ x_{ZI}(t_{ZI}) \end{pmatrix} .
\tag{14}
$$

Let $\delta_3 = s^3 - b(\alpha a_{AB} + \gamma a_{ZI})\, s.$ (15)

Then

$$
\begin{aligned}
\tilde{y} &= \frac{1}{\delta_3}\left\{ \left[\tilde{r}_y + y(t_{ZI})\right] s^2 - \left[x_{ZI}(t_{ZI})\,a_{ZI} + x_{AB}(t_{ZI})\,a_{AB}\right] s \right\} \\
\tilde{x}_{AB} &= \frac{1}{\delta_3}\Big\{ x_{AB}(t_{ZI})\,s^2 + x_{ZI}(t_{ZI})\,\alpha\, b\, a_{ZI} - x_{AB}(t_{ZI})\,\gamma\, b\, a_{ZI} \\
&\qquad - \left[\tilde{r}_y + y(t_{ZI})\right] \alpha\, b\, s \Big\} \\
\tilde{x}_{ZI} &= \frac{1}{\delta_3}\Big\{ x_{ZI}(t_{ZI})\,s^2 + x_{AB}(t_{ZI})\,\gamma\, b\, a_{AB} - \left[\tilde{r}_y + y(t_{ZI})\right] \gamma\, b\, s \\
&\qquad - x_{ZI}(t_{ZI})\,\alpha\, b\, a_{AB} \Big\} .
\end{aligned}
\tag{16}
$$

Transforming back to the time domain

$$
\begin{aligned}
y(t) &= y(t_{ZI}) \cosh \sqrt{(\alpha\, a_{AB} + \gamma\, a_{ZI})\, b}\;\, t \\
&\quad - \frac{\left[x_{ZI}(t_{ZI})\,a_{ZI} + x_{AB}(t_{ZI})\,a_{AB}\right]}{\sqrt{(\alpha\, a_{AB} + \gamma\, a_{ZI})\, b}} \sinh \sqrt{(\alpha\, a_{AB} + \gamma\, a_{ZI})\, b}\;\, t \\
&\quad + L^{-1}\left\{ \frac{\tilde{r}_y\, s^2}{\delta_3} \right\}
\end{aligned}
$$

$$x_{AB}(t) = x_{AB}(t_{ZI}) \cosh \sqrt{(\alpha\ a_{AB} + \gamma\ a_{ZI})\ b}\ \ t$$

$$+ \frac{a_{ZI}\left[\alpha\ x_{ZI}(t_{ZI}) - \gamma\ x_{AB}(t_{ZI})\right]}{(\alpha\ a_{AB} + \gamma\ a_{ZI})} \left\{\cosh \sqrt{(\alpha\ a_{AB} + \gamma\ a_{ZI})\ b}\ \ t - 1\right\}$$

$$- \sqrt{\frac{b}{(\alpha\ a_{AB} + \gamma\ a_{ZI})}}\ \alpha\ y(t_{ZI}) \sinh \sqrt{(\alpha\ a_{AB} + \gamma\ a_{ZI})\ b}\ \ t$$

$$- L^{-1} \left\{\frac{\alpha\ b\ r_{\tilde{y}}\ s}{\delta_3}\right\}$$

$$x_{ZI}(t) = x_{ZI}(t_{ZI}) \cosh \sqrt{(\alpha\ a_{AB} + \gamma\ a_{ZI})\ b}\ \ t$$

$$+ \frac{a_{ZI}\left[\alpha\ x_{ZI}(t_{ZI}) - \gamma\ x_{AB}(t_{ZI})\right]}{(\alpha\ a_{AB} + \gamma\ a_{ZI})} \left\{\cosh \sqrt{(\alpha\ a_{AB} + \gamma\ a_{ZI})\ b}\ \ t - 1\right\}$$

$$- \sqrt{\frac{b}{(\alpha\ a_{AB} + \gamma\ a_{ZI})}}\ \gamma\ y(t_{ZI})\ \sinh \sqrt{(\alpha\ a_{AB} + \gamma\ a_{ZI})\ b}\ \ t$$

$$- L^{-1} \left\{\frac{\gamma\ b\ r_{\tilde{y}}\ s}{\delta_3}\right\}; \quad t_{ZI} \leq t. \tag{17}$$

Figure 2 summarizes equations (4), (11) and (17).

Generalizations of the equations shown in Figure 2 can be made by assuming an arbitrary number of bases utilized by x in the deployment of limited war forces throughout the world. Also y can project his forces into more than one terminal access area at any given time period and in such a manner as to cause x the inconvenience of committing forces to the wrong access area (spoofing). In this way y can achieve a terminal access area objective with a minimum of resources. To handle such problems, the model developed above can be extended to include n bases for deployment of x forces, i.e., $x_1, x_2 \ldots x_n$, and m possible terminal access areas available to y, i.e., $y_1, y_2 \ldots y_m$. In order to simplify the resulting differential equations, we will assume that x will deploy forces to each y_j; $j = 1$ to m from a given subset of the n available bases where no base will deploy forces to more than one access area. If more than one access area is being countered by one of x bases, then a different base at the same geographic location can be defined for each terminal

	$0 \le t < t_{AB}$	$t_{AB} \le t < t_{ZI}$	$t_{ZI} \le t$
$y(t)$	$\int_0^t r_y(\xi)\, d\xi$	$y(t_{AB}) \cosh \sqrt{a_{AB} b}\; t - \sqrt{\frac{a_{AB}}{b}}\, x_{AB_0} \sinh \sqrt{a_{AB} b}\; t + L^{-1}\left\{\frac{\tilde{r}_y s}{\delta_2}\right\}$	$y(t_{ZI}) \cosh \sqrt{(\alpha a_{AB} + \gamma a_{ZI}) b}\; t - \left\{ \left[x_{ZI}(t_{ZI}) a_{ZI} + x_{AB}(t_{ZI}) a_{AB}\right] \cdot \sinh \sqrt{(\alpha a_{AB} + \gamma a_{ZI}) b}\; t \Big/ \sqrt{(\alpha a_{AB} + \gamma a_{ZI})} \right\} - L^{-1}\left\{\frac{\tilde{r}_y s^2}{\delta_3}\right\}$
$x_{AB}(t)$	x_{AB_0}	$x_{AB_0} \cosh \sqrt{a_{AB} b}\; t - \sqrt{\frac{b}{a_{AB}}}\, y(t_{AB}) \sinh \sqrt{a_{AB} b}\; t - L^{-1}\left\{\frac{b\, \tilde{r}_y}{\delta_2}\right\}$	$x_{AB}(t_{ZI}) \cosh \sqrt{(\alpha a_{AB} + \gamma a_{ZI}) b}\; t + \frac{a_{ZI}\left[\alpha\, x_{ZI}(t_{ZI}) - \gamma\, x_{AB}(t_{ZI})\right]}{(\alpha a_{AB} + \gamma a_{ZI})} \cdot \left\{\cosh \sqrt{(\alpha a_{AB} + \gamma a_{ZI}) b}\; t - 1\right\} - \sqrt{\frac{b}{(\alpha a_{AB} + \gamma a_{ZI})}}\, \alpha y(t_{ZI}) \sinh \sqrt{(\alpha a_{AB} + \gamma a_{ZI}) b}\; t - L^{-1}\left\{\frac{\alpha\, b\, \tilde{r}_y s}{\delta_3}\right\}$
$x_{ZI}(t)$	x_{ZI_0}	x_{ZI_0}	$x_{ZI}(t_{ZI}) \cosh \sqrt{(\alpha a_{AB} + \gamma a_{ZI}) b}\; t + \frac{a_{AB}\left[\gamma\, x_{AB}(t_{ZI}) - \alpha\, x_{ZI}(t_{ZI})\right]}{(\alpha a_{AB} + \gamma a_{ZI})} \cdot \left\{\cosh \sqrt{(\alpha a_{AB} + \gamma a_{ZI}) b}\; t - 1\right\} - \sqrt{\frac{b}{(\alpha a_{AB} + \gamma a_{ZI})}}\, \gamma y(t_{ZI}) \sinh \sqrt{(\alpha a_{AB} + \gamma a_{ZI}) b}\; t - L^{-1}\left\{\frac{\gamma\, b\, \tilde{r}_y s}{\delta_3}\right\}$

Figure 2. SUMMARY OF A MATHEMATICAL MODEL FOR A STRATEGIC BASING CONFIGURATION FOR LIMITED WAR FORCES ASSUMING ONLY ONE CONFLICT AREA, ONE ADVANCED BASE AND ONE BASE IN THE ZONE OF INTERIOR.

access being countered. This not too constraining assumption will allow us to uncouple each of y_j's attrition rates leaving these rates a function of some subset of x's n bases.

D. A NEW INTERPRETATION OF THE ASSAULT MODEL

With the statement of the problem of strategic basing of limited war forces in the previous sections of this study and the construct n of a mathematical model to reflect the real world problem in an abstract manner, one cannot help but notice the similarity of the equations summarized in Figure 2 and the assault model developed in Volume I of this study. To illustrate this similarity, let us develop a scenario based upon the equations of Figure 2, only in the format of the assaul model of Volume I.

Let y with an attrition rate b invade a terminal access area (TA) uniformly in time. Assume the existence of native resistance to y as y proceeds to occupy more and more of the terminal access area as a function of time (e.g., as measured in days). Let x have a vital interest in supporting the natives in their resistance to y. To accomplish x's support his forces are prepositioned within the defined theater of operation (AB) and have available forces in a state of readiness at home (ZI).

Allow more than one theater of operation (AB) and if x prepositions his forces at advance bases, assume this allocation of limited war forces must be divided equally amongst all the theaters of operation. Given the order, x's forces with an attrition rate, a can respond to y's invasion after a given time which can be functionally related to the state of readiness of x's forces, the speed of transp tation available to move his forces, and the distance of the terminal access area to the location of x's preposi oned and/or combat-ready forces. Based upon this simple scenario, one can ask the following questions:

How much of his total force should x preposition in the theater of operations adjacent to the terminal access areas vulnerable to invasion?

What response time to y's thrust should x use in conjunction with the allocation of forces both at the advance base (AB) and the zone of the interior (ZI)?

What effect does the attrition rates a and b have upon x's allocation of forces?

Given a native resistance in the terminal access area, at what rate and in what quantity should y plan to inject forces into the terminal access area in order to achieve success against not only the natives but x as well?

What constitutes success (or failure) for each side, and what type of pay-off function adequately measures each side's objectives?

Note that the scenario described above does not define a realistic allocation of forces for y. One can give y an allocation by assuming a certain percentage of his forces are projected into the terminal access area at the time the invasion is initiated and the rest of y's forces following at a constant daily rate until y's total force has been deployed. If we let the percentage of y's force instantly deployed run the gamut of possibilities (i.e., from 0 to 100%), then a decision surface can be generated. The problem with y's allocation decision is that intuitively one can see no disadvantage to the deployment of all of y's force when the invasion is initiated. However, from a constraint point of view, y might not be able to deploy all his forces immediately and therefore, it would be of interest to determine if any minimum rates of deployment for y exist in order to achieve a successful objective. As for x, there are both advantages and disadvantages in the spectrum of allocation choices and a realistic game-theoretic solution exists.

As for the pay-off for both sides, one would be tempted to use the same pay-offs defined in the Assault Model of Volume I. That is, the time length of battle (in this case the time length of campaign would be appropriate) and the number of excess survivors of the winning side for a given threshhold of survivors for the losing side. The time length of battle criteria would be more applicable to the weaker side, whereas, the number of excess survivors would be a more appropriate measure of success for the stronger side.

Figure 3 summarizes a standard case for the above defined scenario and Figure 4 yields the results of a machine run of this scenario using the assault model. The columns of Figure 4 represent the allocation of x to the advance base (AB) starting with the 100% allocation on the left. Above these columns are shown the number of forces actually deployed into battle

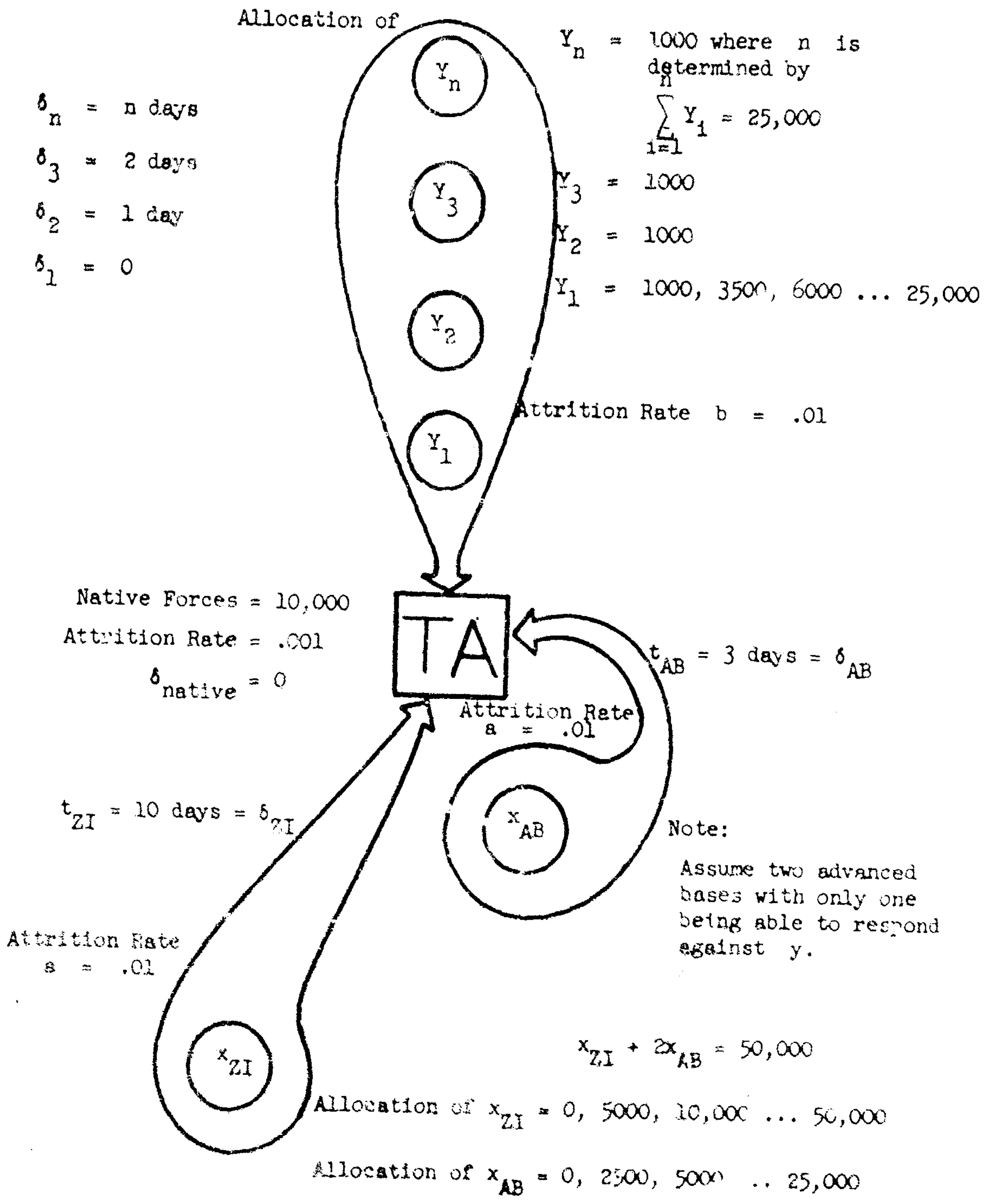

Figure 3. SINGLE INVASION SCENARIO BASED UPON THE ASSAULT MODEL OF VOLUME I

Figure 4. NUMBER OF x SURVIVORS AND LENGTH OF CAMPAIGN IN DAYS FOR THE y INVASION ILLUSTRATED IN FIGURE 3.

(35000)	(37500)	(40000)	(42500)	(45000)	(47500)	(50000)	(52500)	(55000)	(57500)	(60000)
					a) Survivors (men)					
21977	26490	30352	33868	37167	40319	43363	46326	49225	52075	54912
21521	26116	30016	33554	36869	40031	43082	46051	48955	51808	54654
21073	25752	29691	33252	36581	39753	42812	45787	48696	51552	54407
20633	25400	29378	32962	36305	39487	42554	45534	38337	51307	54170
20202	25060	29077	32682	36040	39232	43206	45291	48209	51073	53944
19466	24732	28788	32415	35786	38988	42069	45060	47982	50849	53729
19370	24417	28511	32145	35544	38755	41843	44840	47705	50636	53525
18969	24115	28247	31916	35315	38535	41624	44630	37560	50433	53331
18581	23626	27997	31686	35100	38329	41420	44429	47364	50240	53141
					b) Length of Campaign (days)					
144	119	104	93	85	79	74	69	65	62	59
147	121	105	94	86	79	74	69	66	62	59
150	122	106	95	86	80	74	70	66	62	59
153	124	107	96	87	80	75	70	66	63	59
156	126	108	96	88	81	75	70	66	63	60
159	127	109	97	88	81	75	71	67	63	60
163	129	110	101	89	82	76	71	67	63	60
166	130	111	98	89	82	77	71	67	64	60
$\boxed{169}$	$\boxed{132}$	$\boxed{112}$	$\boxed{99}$	$\boxed{90}$	$\boxed{83}$	$\boxed{77}$	$\boxed{72}$	$\boxed{68}$	$\boxed{64}$	$\boxed{60}$
					c) x's Casualties					
13223	11010	9648	8632	7833	7181	6637	6174	5775	5425	5088
13479	11384	9984	8946	8131	7469	6918	6449	6045	5692	5346
13927	11748	10309	9248	8419	7747	7188	6713	6304	3448	5593
14367	12100	10622	9538	8695	8013	7446	6966	6553	6193	5830
14798	12440	10923	9818	8960	8263	7694	7209	6791	6427	6056
15534	12768	11242	10085	9214	8512	7931	7440	7018	6651	6271
15630	13083	11489	10355	9456	8745	8157	7660	7235	6864	6475
16031	13385	11753	10584	9685	8965	8376	7870	7440	7067	6669
$\boxed{16419}$	$\boxed{13674}$	$\boxed{12003}$	$\boxed{10814}$	$\boxed{9900}$	$\boxed{9171}$	$\boxed{8580}$	$\boxed{8071}$	$\boxed{7636}$	$\boxed{7260}$	6859

by x (this includes native resistance). Since this number is a variable, (Figure 4c) was included to indicate casualties rather than survivors because it was felt that the casualty pay-off to x would be more meaningful. The rows of these matrices (Figure 4) represent y's invasion of the terminal access. The first row indicates the result of a linear flow of forces (1000 men/day). The second row indicates the same linear flow of troops, however, the first day 2,000 men initiate the invasion. The third row again indicates 1,000 men/day flow of forces, however, the first day is increased to 3,000 men. This process is repeated for nine rows.

The optimum (min-max = max-min) strategy for both sides is in the lower right-hand corner of Figure 4b and 4c. That is x should station his forces in the zone of the interior (ZI) and y should inject as many of his forces into the invasion the first day as possible. What is even more important is that the surfaces (both casualties and campaign time) contain no decision thresholds which are different from the basic strategies.* Also, x's response time is insensitive to the pay-off as defined by length of campaign. However, x's total number of forces deployed to the terminal access area influenced the optimum pay-off considerably. One questions the wisdom of stationing forces overseas for the purpose of responding to a single threat if these forces must be spread out over many theaters of operations. However, the real world contains many terminal access areas as potential limited war threats and splitting forces between overseas theaters of operations (AB) and the zone of interior (ZI) may have a rationale. The purpose of this study then is to determine under what conditions the stationing of limited war forces at advanced bases overseas becomes desirable if the prime purpose of such deployment is to counter enemy thrusts into many potential terminal access areas over the earth.

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* See Appendix A for the definition of decision thresholds.

E. THE STRATEGIC PROBLEM IN BASING LIMITED WAR FORCES

1. The Recommitment Problem

The results of the last section illustrate the problem in its basic simplicity. For a single terminal access area conflict, the trade-off in having "quick" response-time resulting in a smaller amount of resources (limited war forces) to be deployed does not justify the stationing of these forces overseas if the defender must spread his forces over many theaters of operations when stationing them away from the zone of the interior. Of course, it can also be assumed that given enough time all the forces available to the defender at advanced bases could be injected into the conflict area, thus removing the disadvantage of precommitting forces to advance bases. This scenario can be abstracted in the following manner:

Blue, defending ten different terminal access areas from possible invasion by red, prepositions part of all of his forces equally to the ten terminal access areas or maintains part or all of his forces in alert status in the zone of interior. The exact allocation of total forces between geographic positions is a strategic decision variable to be optimized by blue. On the other hand red chooses from a uniform distribution in a random manner* one of the ten terminal access areas to invade and allocates his forces into two groups. One group is to directly engage blue's prepositioned force. The other group is to prevent blue from reinforcing his prepositioned forces by a cutoff action. This allocation of total forces between direct combat with blue and a cutoff action represents red's strategic decision variable. A standard configuration for the above scenario is illustrated in Figure 5. Figure 6 gives the input parameters for use in the Assault Model. Two payoffs were computed in this scenario: the number of blue survivors when red's troops are reduced to zero (B - R) and the length of time (in days) of the campaign t. The parameter that was varied in this particular scenario was the order

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*If blue defends all the terminal access areas equally, blue is assuming that red will attack any of the areas with equal probability. Any variance from this strategy on red's part can be treated as another parameter in the scenario.

THE DEFENSE OF TEN TERMINAL ACCESS AREAS ASSUMING THE RECOMMITMENT OF ADVANCE BASE FORCES TO THE INVASION AREA

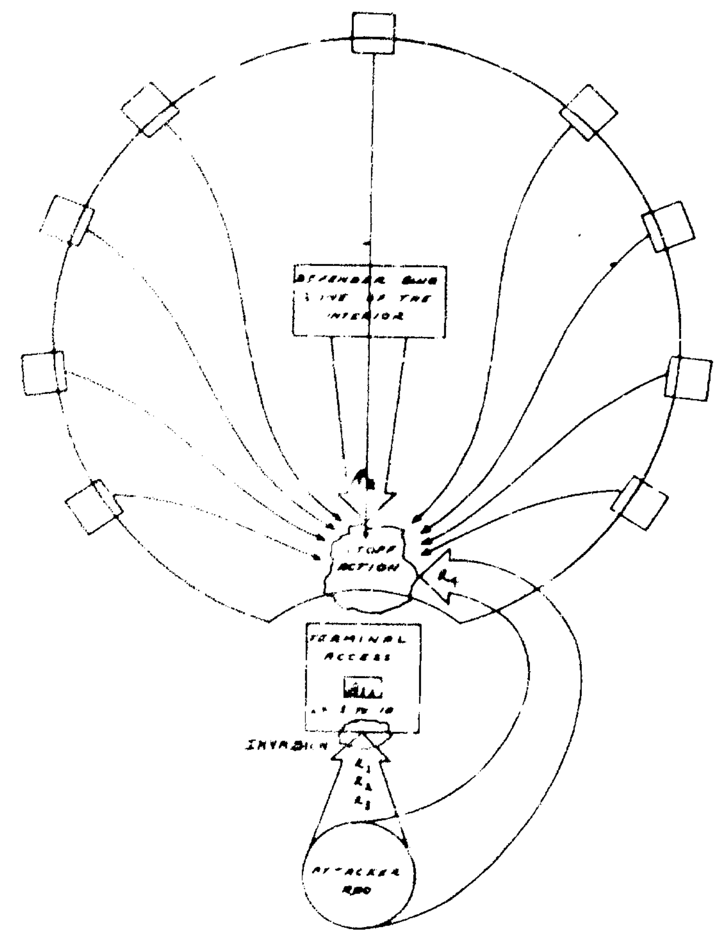


Figure 5

STRATEGIC VARIABLE

UNIT	TROOP ALLOCATION (No. of Fighting Men)	Δt (Days)	t_d (Days)	ATTRITION Men Kld. / Day / Fitng Men	1st LEVEL OPPONENTS	2nd LEVEL OPPONENTS
B_{11}		0	1.5	.4	R_1, R_2, R_3	R_4
B_{12}		1.5	1.5	.4	R_4	R_1, R_2, R_3
B_{13}		1.5	1.5	.4	R_4	R_1, R_2, R_3
B_{14}		3.0	1.5	.4	R_4	R_1, R_2, R_3
B_{15}	$0 \le B_{1i} \le 5000$ $i = 1$ to 10	3.0	1.5	.4	R_4	R_1, R_2, R_3
B_{16}		4.5	1.5	.4	R_4	R_1, R_2, R_3
B_{17}		4.5	1.5	.4	R_4	R_1, R_2, R_3
B_{18}		6.0	1.5	.4	R_4	R_1, R_2, R_3
B_{19}		6.0	1.5	.4	R_4	R_1, R_2, R_3
B_{110}		7.5	1.5	.4	R_4	R_1, R_2, R_3
B_2	$0 \le B_2 \le 50{,}000$	7.0	1.5	.4	R_4	R_1, R_2, R_3
R_1		0		.1		
R_2	$0 \le R_2 \le x/3$ $i = 1 - 3$ $x = 0$ to $50{,}000$	.5	1.5	.2	B_{11}	B_2, B_{1i} $i = 2$ to 10
R_3		1.0		.4		
R_4	$0 \le R_4 \le x$ $x = 0$ to $50{,}000$	0	~	.15	B_2, B_{1i} $i = 2$ to 10	None

Figure 6. INPUT PARAMETERS TO THE RECOMMITMENT SCENARIO OF FIGURE 5.

of battle (number of troops) ratio between the both sides R/B; $\frac{10,000}{50,000} \leq R/B \leq \frac{50,000}{50,000}$.

Figure 7 gives the complete solution to this scenario as a function of the order of battle ratio parameter (R/B) and defined payoffs (B - R) and t. The first column on the left illustrates the unrestricted play of the game for both blue (the defender) and red (the attacker) in terms of blue survivors (blue maximum-minimizes and red minimum-maximizes), utilyzing the min-max = max-min criteria if applicable. The basic strategy for both sides using the length of campaign t payoff (Figure 7 third column) is differentiated by circles '⊙' and crosses 'X' depending upon whether blue attempts to guarantee in a game-theoretic sense a minimum or maximum length of campaign time t.

The results of Figure 7 indicate that red, the attacker using the (B - R) criteria, initiates the invasion of the terminal access area (TA) with all his forces prepared for immediate combat against blue's precommitted forces. On the other hand blue, the defender, will in general deploy in the zone of the interior (ZI) until red's invasion has been initiated and then deploy against red after seven days (Figure 6). This period of time represents the embarkation, transportation, and final assault time for blue's forces. The second column of Figure 7 indicates the constraints on blue, the defender, in playing optimally. These constraints are based upon the natural discontinuity levels of the blue-red decision (payoff) surface as defined in Volume I of this study. The second chart from the top in column two of Figure 7 indicates that blue must keep at most over 90% and at least over 60% of his forces deployed in the ZI in order that the above unrestricted strategies apply. The variation in this threshold seems to be a linear function of the size of the invading red's forces relative to blue (R/B). The higher the threshold the smaller the invading force. When the invading force equals the defending force (R/B = 1), no threshold exists. This means blue's strategy is to always deploy as many of his forces as possible from ZI. For other values of (R/B ≠ 1), blue's constrained

THE RECOMMITMENT SCENARIO

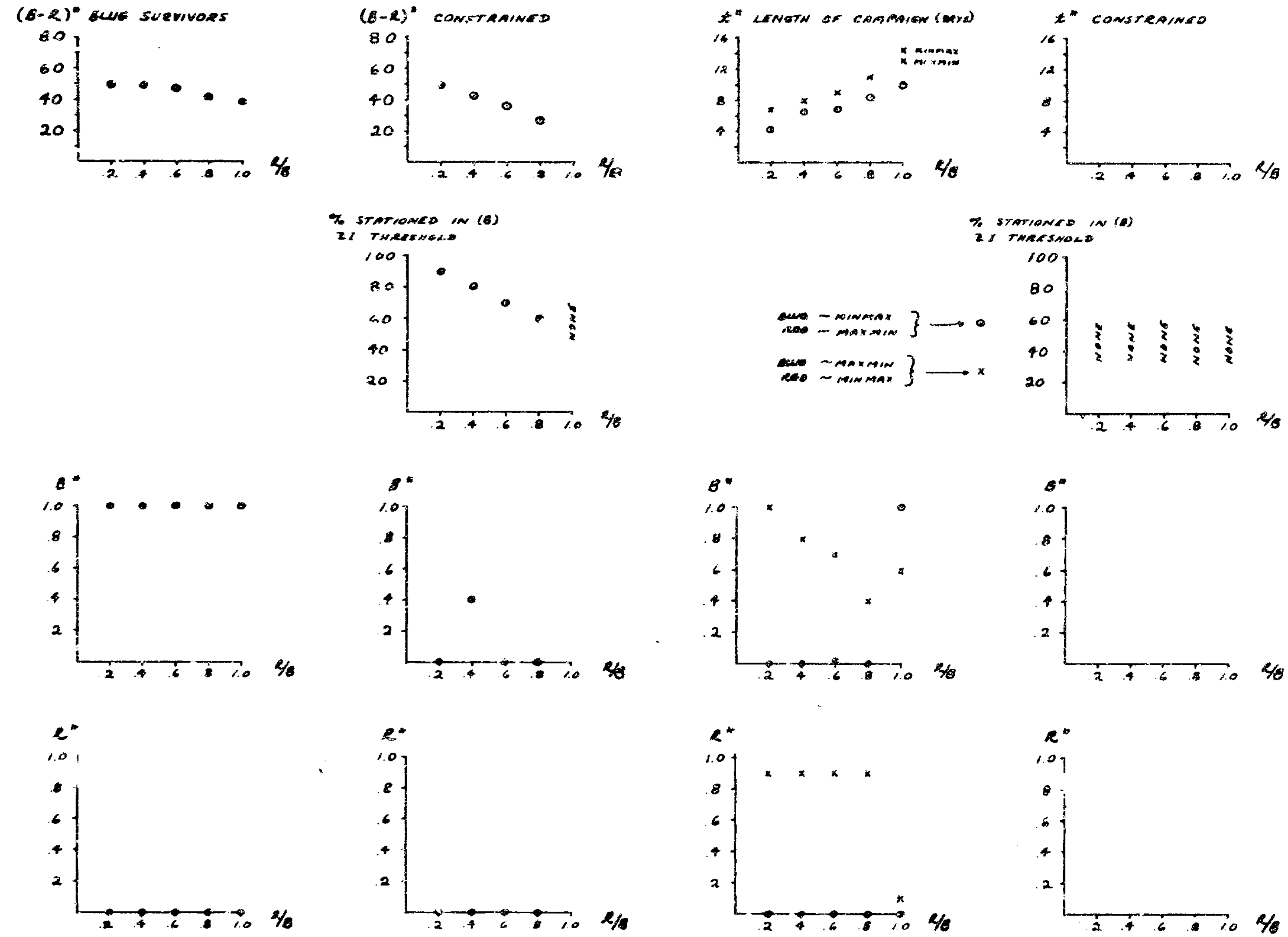


Figure 7

strategy changes basically to committing his forces to the TA before the invasion if his threshold ZI commitment is not exceeded (see second and third chart from the top of the second column of Figure 7). Summarizing the policy maker's decision criteria for blue seems to be that the stronger the invading threat (red), the less constrained the defender (blue) is in keeping his forces in the ZI when playing optimally (i.e., deployment from ZI). This rule is demonstrated in its extreme in the case of $R/B = 1$ where no threshold exists. That is no matter how many of blue's forces are constrained to be stationed in the various overseas TA's, the rest should be deployed from the ZI in order to insure optimality in allocation. In other words, for the case of $R/B = 1$, there is never a "flip-flop" optimum decision change from ZI to TA force deployment based upon a natural discontinuity level (threshold) of the decision surface as there is for values of $R/B \neq 1$.

The analysis of the strategies based upon the length of campaign t yields a completely different picture (third column, Figure 7). First of all, there are no natural discontinuity levels in the decision surface (fourth column, Figure 7). Red, the invader's optimal strategy is basically to attack the TA if attempting to maximize (in a game-theoretic sense) the length of campaign t and deploy his forces in a cutoff operation if attempting to minimize the campaign time. For $R/B < 1$, where red loses the campaign, the former payoff would be sensible for red. When $R/B = 1$, red almost obtains parity with blue and the latter payoff breaks down into a game-theoretic mixed strategy (for red minimizing) indicating that both sides better keep their strategies a secret. This is done by both sides randomizing their strategic variables according to a distribution law developed from a generalization of the min-max = max-min payoff criteria. For requirements analysis purposes (as opposed to the active evaluation of specific scenarios), mixed strategies are not too meaningful except at the point at which they go from pure to mixed. The reason for this is that the analyst would never knowingly state a system requirement without knowing the outcome to be guaranteed by such a requirement.

Blue, the defender, which in this case is the superior force, would attempt to minimize the length of campaign, plays optimally by defending the TA's directly and recommitting all forces to the TA being invaded according to the time schedules shown in Figure 6. The exception is when R/B = 1 and parity between the sides almost occurs, blue's optimal strategy reverts to ZI deployment.

If on the other hand blue attempts to maximize the campaign length t, because this scenario represents the initial disposition of forces in a continuing campaign, blue's strategy is very definitely tied to a specific allocation of forces between the ZI and TA which is a function of the time delays defined in Figure 6. The second chart from the bottom of the third column of Figure 7 indicates that as the intensity of threat increases the allocation of total forces shifts in favor of the terminal access areas.

Summarizing, this analysis of a rather artificial scenario indicates significant decision levels and corresponding thresholds exist which are inherent to the problem. Factors of space, time, and forces available as delineated in Figure 6 have direct functional relationships with respect to the structure of the problem as defined by the specific payoffs used. Because of this, the analysis of a realistic scenario which will define specific space and time characteristics seems appropriate. This will be the subject matter of the next section.

2. A War in Southeast Asia

a. Introduction

If blue, the defender, maintains control of the seas, then red, the attacker, is limited to expansionist activities in the peripheral terminal access areas only. The scenario illustrated in Figure 8 gives a significant real world situation which can be used as the basis for the analysis of the logistic characteristics of the strategic basing problem for limited war forces of both the United States of America (blue the defender) and the Chinese Mainland (red the attacker). Three general terminal access areas can easily be defined for red. They are: (1) India/Burma; (2) Korea; and (3) the general area defined by Thailand/Laos/Cambodia/South Vietnam. Blue can counter such a threat in two ways. The first by resisting the invasion of the above defined terminal access areas by the projection of troops from

A WAR IN SOUTHEAST ASIA

Figure 8

the ZI and/or an advanced base such as Hawaii, Formosa, Okinawa, Philippines, Japan, Korea and South Vietnam. These latter two advanced bases are also terminal access areas. The second method of resisting red's peripheral expansion would be to directly attack red's homeland and force red to recommit her forces in the defense of the homeland. The great circle distances between the nations involved in this scenario are shown in Figure 9. The large distances involved in this scenario are directly related to the magnitude of the logistics problems facing blue, the defender.

> Just where should blue station its forces (ZI or AB)?
>
> In what state of readiness should blue be when red initiates the attack against TA's No. 1, No. 2, and No. 3?
>
> And what level of fire power relative to red should blue use in countering the attack?

are questions to be answered by analyzing the above scenario.

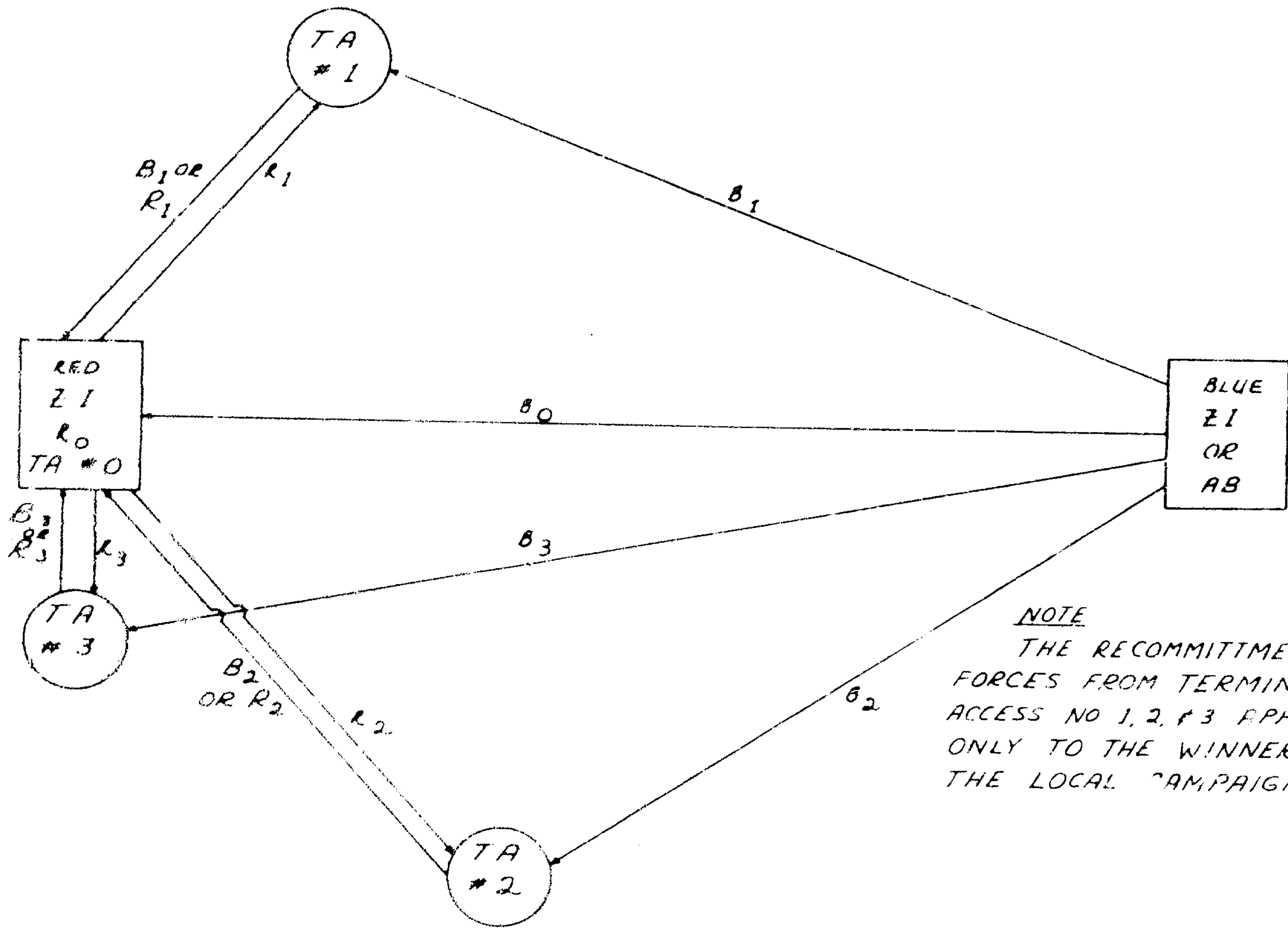
b. A Standard Case

Figure 10 represents an idealization of the Southeast Asia War Scenario with the arrows indicating the logical projection (both initial deployment and recommitment) of forces for both sides as a function of time. Again the Assault Model of Volume I will be used. Figure 11 indicates the input parameters of the Assault Model and the corresponding interpretation of these parameters from the logistic point of view. The normalized attrition constant 'a' has been redefined as <u>casualties</u> per opposing force per <u>campaign time</u>. Thus we can talk about the defeated side having 100% casualties if we define a casualty as a member of the force that can no longer fight. In this way the Assault Model of Volume I will still be applicable only with Lanchester's differential law of combat being applied to the fighting over the total theater of operations rather than just over a particular battlefield. The basic value of the normalized attrition constant 'a' used for both sides (blue and red) is .001. The derivation of this number is based upon the assumption that the numbers of troops committed to a campaign of the type defined above by both sides is usually of the same order of magnitude (Reference 2). Therefore, the ratio of casualties to opposing forces of either side is of the order of magnitude of one. Then what remains of the normalized attrition constant

			ZI		HAWAII	SOUTH KOREA	OKIN-AWA	FOR-MOSA	PHILI-PPINES et al	SOUTH VIET-NAM	INDIA	CHINA
			NY	CAL								
	ZI	NY										
	ZI	CAL	2400									
dvanced Base for Southeast Asia	HAWAII		4800	2400								
Terminal Access No. 1	SO. KOREA		7200	5920	4800							
Advanced Base for Korea and China	OKINAWA		7400	6240	4960	640						
	FORMOSA		8000	6400	5120	960	480					
Advanced Base for So. Vietnam and India	PHILIPPINES		8640	7040	5200	1600	960	960				
Terminal Access No. 2	SO. VIETNAM et al		8600	784[illegible]	6080	1920	1440	960	1120			
Terminal Access No. 3	INDIA		7680	8160	7680	3200	3040	2560	3040	1920		
Terminal Access No. 0	CHINA		7360	6880	5600	1120	1120	640	1600	1120	1920	

Figure 9 GREAT CIRCLE DISTANCES (IN STATUTE MILES) PERTINENT TO A WAR IN SOUTHEAST ASIA.

IDEALIZATION OF SOUTHEAST ASIA WAR SCENARIO



NOTE
THE RECOMMITTMENT OF FORCES FROM TERMINAL ACCESS NO 1, 2, & 3 APPLIES ONLY TO THE WINNER OF THE LOCAL CAMPAIGN.

Figure 10

'a' is the inverse of the campaign time (in days). In recent years (this century) conventional wars of the magnitude inherent to the Southeast Asia scenario postulated above have lasted from three to five years which converts to a normalized attrition constant 'a' of the order of magnitude of one-thousandth [i.e. .001 in units of $(\text{days})^{-1}$] or order of magnitude O (.001) in days. The reader should not worry about the accuracy of this estimate of 'a' since the results of the following analysis will not be dependent upon this absolute value used. As one can see in Figure 11, blue, the defender, will trade-off his inferiority in order of battle ratio (i.e., $B/R \leq .5$) in part by deploying his forces with greater firepower (by orders of magnitude $.001 \leq a \leq 1.0$) in order to achieve the necessary superiority. The question as to when blue's excess firepower relative to red compensates for the low order of battle ratio is more important than the determination of either side's absolute value of the normalized attrition constant. Once blue's factor of superiority relative to red is known, then other models will have to convert such information into battlefield strategy and tactics (Reference 1, section II, E). Of course, there are other trade-off factors available to blue to achieve campaign objectives. They are shown in Figure 11 as time delays Δt and t_d, which when interpreted in light of the Southeast Asia scenario represent deployment and recommitment times of both side's forces over the various terminal access areas. Again the derivation of the numbers shown in Figure 11 does not influence the future analysis. What really is important is the overall sensitivity of these mobility factors (Δt and t_d) to the objectives of each side during and after the campaign. Thus it should be possible to determine the contribution of readiness of troops, embarkation and movement time, assault time, and recommitment time towards each side achieving its objectives. As an example, these parameters might be allowed to vary in such a manner as to describe the characteristics of certain transport devices such as troop transport (e.g., 10 knots), hydrofoil (e.g., 50 knots), and/or jet aircraft (e.g., 600 knots) in order to determine their individual contribution to either side's success. A similar statement can be made about any other parametric input to the model which reflects a real world attribute (e.g., order of battle ratio).

Figure 11 THE INPUT PARAMETERS TO THE ASSAULT MODEL AND THE CORRESPONDING INTERPRETATION OF THESE PARAMETERS FROM THE LOGISTIC POINT OF VIEW

Participant		Troop Allocation	Δt (days)	t_d (days)	Normalized Attrition Constant 'a' Casualties Per Opposing Force Per Campaign Time	1st Level	2nd Level
Deploy Loca-tion	Unit Desig-nators	No. of Troops	Embark. Readiness & Movement Assault	Recommit-ment		Opponent	Opponent
		$\sum_{i=0}^{3} R_i = 2 X 10^6$					
TA #0	R_0	$0 \le R_0 \le 2X10^6$	0	~	.001	B_0	~
TA #1	R_1	$\frac{2 X 10^6 - R_0}{3}$ each	0^*	60 ≠	.001	B_1	B_0
TA #2	R_2		0^*	60 ≠	.001	B_2	B_0
TA #3	R_3		0^*	90 ≠	.001	B_3	B_0
		$\sum_{i=0}^{3} B_i = B$ where B=.1, .4, .7, 1.0X.0^6	(see distance table below)				
ZI and/or for TA#0	B_0	$0 \le B_0 \le B$	$AB = 15^{**} + \frac{640 \#}{v}$ $ZI = 15^{**} + \frac{6880 \#}{v}$	~		R_0	~
TA #1	B_1	$\frac{B - B_0}{3}$ each	$AB = 0^{x}$ $ZI = 15^{**} + \frac{5920 \#}{v}$	75 ≠	$.001 \le a \le 1.0$	R_1	R_0
TA #2	B_2		$AB = 0^{x}$ $ZI = 15^{**} + \frac{7840 \#}{v}$	75 ≠		R_2	R_0
AB for TA #3	B_3		$AB = 15^{**} + \frac{3040}{v}$ $ZI = 15^{**} + \frac{8160}{v}$	75 ≠		R_3	R_0

st.mi	TA#0		TA#1		TA#2		TA#3	
day	AB	ZI	AB	ZI	AB	ZI	AB	ZI
240	18	45	0	40	0	48	28	49
1200	16	21	0	20	0	22	18	22
Jet	15	16	0	16	0	16	15	16

SEE NEXT PAGE FOR FOOTNOTES

FOOTNOTES FOR FIGURE 11

See Figure 9

≢ TA →ZI = 30 days, ZI → TA #0 = 45 days (a param : to be varied).

≮ Practically a guess but seems reasonable.

X The advance base AB .d terminal access TA #2 and #3 are the same.

** 15 day fixed deployment time component based upon readiness = 0, embarkation = 10 days, and assault = 5 days.

* $\Delta t = 0$ represents red's capability to initiate the invasion of the three terminal access areas (TA) simultaneously.

Figure 12 indicates the basic play of the variable parameter for the standard case is the number of troops in blue's force. This value varies from 100,000 to 1,000,000 men while red utilizes a constant number of 2,000,000 men. Both sides allocate all their forces either to the three terminal access areas equally (TA No. 1, 2, 3) and whatever is left over to the Chinese homeland (TA No. 0). This allocation between TA No. 0 and TA No. 1, 2, 3 is done in 10% intervals of total forces (see Reference 1) and is called the strategic variables available to both sides. Blue's partial or complete allocation to TA No. 0 denotes his intention to resist red's invasion of TA No. 1, 2, and 3 by forcing red to defend his homeland, if red initiates an invasion of TA No. 1, 2, and 3. Red recognizing the possibility of blue's defensive strategy to attack TA No. 0 must decide how much, if any, of his forces must be allocated to TA No. 0 for possible defensive combat. Both allocations are strategic variables and will be chosen using the game theoretic criteria similar to that used in Volume I of this study. Two payoffs are available in this scenario. They are numbers of forces that survive (do not become a casualty) when one side has 100% casualties (in the sense defined above) and the total time of campaign in days. In this latter payoff the victorious side will attempt to guarantee a minimum time and the loser will guarantee a maximum time. The reasons for this were explained in the analysis of the recommitment scenario (section E, 1).

The results of the game as defined by the inputs in Figure 12 are shown in Figure 13. The first column denotes the unrestricted play of the game where pure strategies exist for both sides with each allocating all or 100% of their forces to TA No. 0 (i.e., $\left\{B^*, R^*\right\} = \left\{0, 0\right\}$ for B: $.1 \times 10^6 \leq B \leq 1.0 \times 10^6$). The length of campaign varies directly with B and goes from 95 days to 594 days (see second graph, first column, Figure 13). This analysis appears to be incompatible with the real world scenario defined above in which red, the attacker, was supposed to initiate the action with a simultaneous invasion of TA No. 1, 2, and 3. Yet in red's optimal play of the game, his allocation should have been to TA No. 0 or defend the homeland against blue's response to the invasion. This dilemma can be rectified, if the analyst constrains red to define his intentions towards TA No. 1, 2, and 3 in terms that can

Figure 12 SOUTHEAST ASIA SCENARIO

STANDARD CASE*

Unit	Troops	Δt	t_d	Attrition	1st Level	2nd Level
R_0	0 to 2×10^6 in 10% intervals	0	0	.001	B_0	~
R_1	$\frac{2 \times 10^6 - R_0}{3}$ each	0	60	.001	B_1	B_0
R_2		0	60	.001	B_2	B_0
R_3		0	90	.001	B_3	B_0
B_0	0 to B where $B = .1 \times 10^6$, $.4 \times 10^6$, $.7 \times 10^6$, 1.0×10^6 in 10% intervals	45	0	.001	R_0	~
B_1	$\frac{B - B_0}{3}$ each	0	75	.001	R_1	R_0
B_2		0	75	.001	R_2	R_0
B_3		28	75	.001	R_3	R_0

*Readiness = 0

Embarkation + Movement + Assault = $15 + \frac{\text{Distance}}{v}$;

v = 240 st.mi./day Direct

Embarkation + Movement + Assault = $(R_1\ R_2\ R_3)$ = (60, 60, 90) Recommit

$(B_1\ B_2\ B_3)$ = (75, 75, 75) Recommit

SOUTHEAST ASIA SCENARIO STANDARD CASE

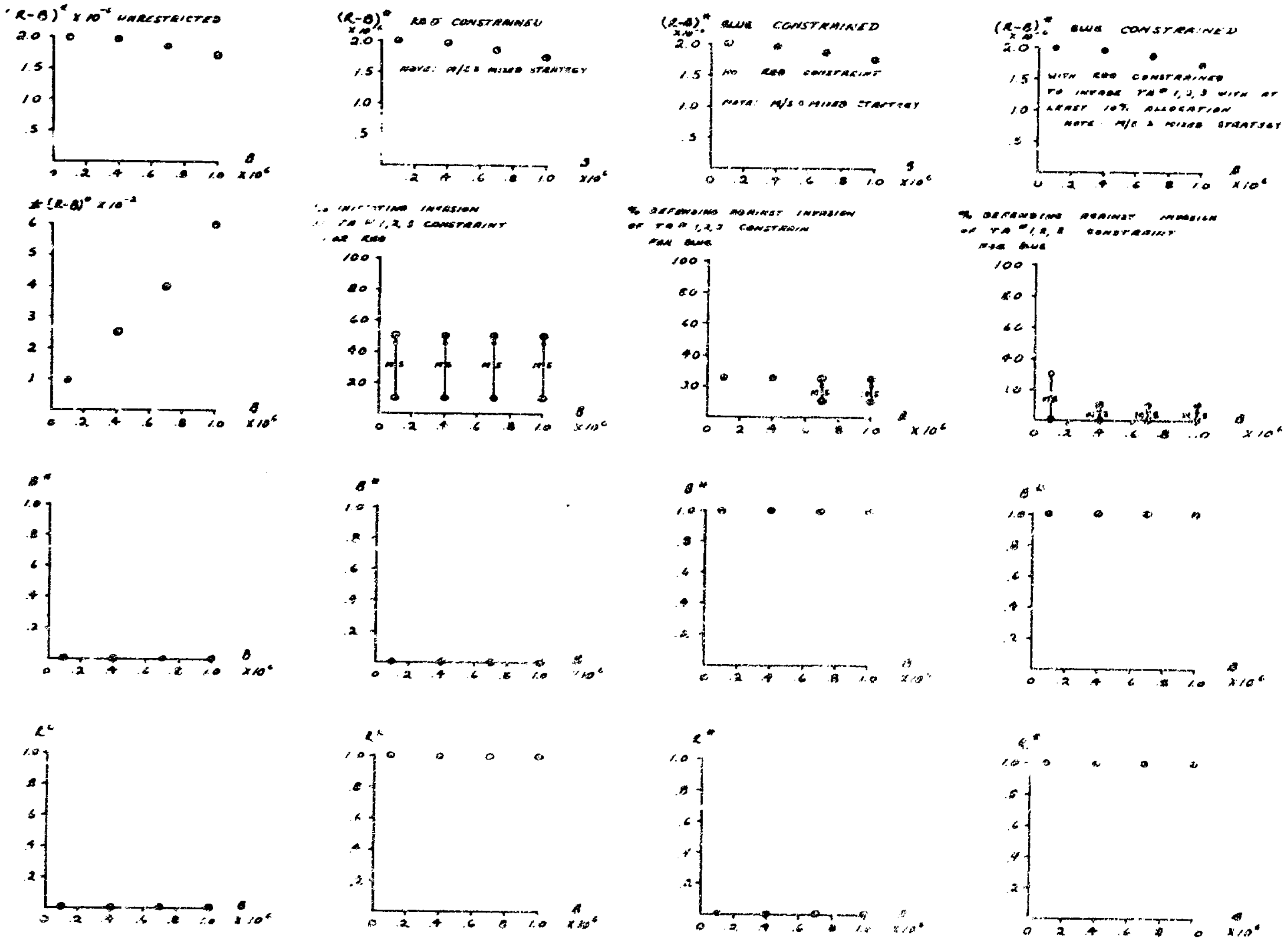


Figure 13

be put into the mathematical model being used. This is accomplished by forcing red to allocate at least 10% of his total forces to TA No. 1, 2, and 3. In the language of the model, red's strategic variable runs from 10% to 100% to TA No. 1, 2, and 3 in ten per cent intervals while blue, the defender, has the unconstrained (free) strategic choice of allocating any part of his total resources to TA No. 1, 2, and 3 while the remainder goes to TA No. 0. The play of this partially constrained game results in a mixed strategy which means both sides secretly randomize their choice of troop allocation in conformance with the solution of the game. However, if red should feel constrained to commit at least 50% of his forces to the invasion of TA No. 1, 2, and 3, then the optimal strategies would revert back to pure strategies with red allocating all its forces to TA No. 1, 2, and 3, and blue allocating all to TA No. 0 (i.e., $\left\{B^*, R^*\right\} = \left\{0, 1\right\}$ for B: $.1 \times 10^6 \leq B \leq 1.0 \times 10^6$). The second column of Figure 13 indicates these results. The per cent initiating the invasion of TA No. 1, 2, and 3 (constraint for red) is indicated by the circle "◯" in the second graph of the second column. Red's constraint band from 10% to 50%, representing a mixed strategy, indicates from the real world point of view indecision on red's part to commit all his troops to TA No. 1, 2, and 3 if constrained to commit up to 50% of his forces in initiating the invasion. However, if 50% or more of his forces, for real world reasons, are committed to invading TA No. 1, 2, and 3, then the optimal strategy for red would be to commit all his forces to TA No. 1, 2, and 3. On the other hand, blue's response strategy would be to attack red at TA No. 0 and force red to return to his homeland to defend himself (see third and fourth graph, second column, Figure 13).

The third column of Figure 13 indicates the natural constraints in blue's decision structure assuming red was not constrained at all. A real world situation which would correspond to this case would be red attempting a deceptive move against blue which would result in blue acting against red without red ever committing forces to TA No. 1, 2, and 3. In strategic warfare language this is called blue's pre-emptive strike.

If blue is constrained to keep at least 30% of his forces overseas at TA No. 1, 2, and 3, then the optimal solution for both sides is: $\{B^*, R^*\} = \{1, 0\}$ for B: $.1 \times 10^6 \leq B \leq 1.0 \times 10^6$. A mixed strategy occurs between 10% and 30% for blue order of battle magnitudes of $.7 \times 10^6 \leq B \leq 1.0 \times 10^6$. For 30% and over, blue strategy is pure, i.e., all forces are allocated to TA No. 1, 2, and 3. For low order of battle ratios, i.e., when R: $.1 \times 10^6 \leq R \leq .4 \times 10^6$, there is a clearcut "flip-flop" decision discontinuity level at 30% which has significance in this study. If blue is <u>at most</u> constrained to keep up to, but not including, 30% of his forces overseas (i.e., less than 30% allocated to TA No. 1, 2, and 3) his optimal strategy is to keep none of his remaining forces overseas; but rather, 100% of his remaining forces allocated to the ZI for offensive action against TA No. 0. In the meantime, red always allocates optimally when defending the homeland (last graph, third column, Figure 13). Since nothing in the model reflects blue's intentions except his constraining allocation of up to but not including 30% of his forces deployed to TA No. 1, 2, and 3, the analyst can understand blue's hesitancy to allocate all his forces to TA No. 1, 2, and 3 <u>as a form of deterrence</u>. Notice this was not true of the red-constrained-strategy of the second column of Figure 13. Once red was committed to a 10% invasion of TA No. 1, 2, and 3, the payoff yielded a mixed strategy up to a 50% invasion constraint, and a pure strategy of full commitment beyond the 50% constraint. Mixed strategy denotes the random possibility of a full commitment which from the real world point of view is similar to pure strategy, though it certainly does not imply a form of deterrence. In the next section, the author will attempt to quantify, from each of the combatants' point of view, the intentions of deterrence, and will continue this line of reasoning in order to achieve a special measure of merit for the design and evaluation of limited war basing configurations.

Finally, the fourth column of Figure 13 indicates blue's natural discontinuity decision level when red is constrained to attack TA No. 1, 2, and 3. At blue's 10% constraint level, the optimal strategy for both sides appears to be the allocation of all their forces to TA No.

1, 2, and 3, (i.e., $\{B^*, R^*\} = \{1, 1\}$ for B: $.4 \times 10^6 \leq B \leq 1.0 \times 10^6$). When $B = .1 \times 10^6$, the constraint threshold goes to 30%. Below both these constraint thresholds (down to the 0% level), mixed strategies are applicable. This case represents the real world situation that exists today in which blue is committed (constrained) to defend against red's invasion (again a constraint) of TA No. 1, 2, and 3. Neither red nor blue has yet made his optimum allocation decisions and the indecision will, in the future, probably result in random choices of full or minimum commitment of forces. However, if blue decides to go over its natural discontinuity threshold level (see second graph, fourth column, Figure 13), both sides will fight it out as a pure strategy at TA No. 1, 2, and 3.

c. Deterrence

> "The ideal situation would be to be able to project just enough limited war forces at sufficient speed to deter the opposition from acting at all."
>
> (Section IIA)

How does one achieve the above "ideal" for blue's (the defender's) limited war system in terms of the following: available strategic basing configurations, the firepower levels technically and politically feasible, the strategic reaction times possible, order of battle, and similar characteristics attributed to red (the attacker)?

The standard case of "A War in Southeast Asia," analyzed in the last section, gave some clues as to how one would go about defining the ideal situation. This case presents both sides with their natural environmental, logistic, and constraining advantages and disadvantages. It assumes equality in firepower (as reflected by the parameter 'a'). Blue, with an inferior number of combatants, constrained by long logistic pipe-lines and inherently defensive intentions, must present its limited war systems characteristics to red in such a credible force posture that red, when acting in a rational manner, will not attack; or if red becomes irrational and does initiate an attack, red's (thru his non-optimal play) will lose the war.

In the last section it was noted that the decision to deploy forces in an aggressive way had a natural discontinuity level in which the military planner, if forced to commit at least a certain percentage of his forces (this percentage being a natural discontinuity of the decision surface), would automatically commit all of his forces. In terms of the Southeast Asia Scenario, is it possible for blue, the defender, to present a force posture such that red, the attacker, would never "go all the way" if red was constrained or forced to "go part of the way"? If blue could present such a force posture, it could be said that blue effectively deterred red, the attacker, provided red was rational. For if red intended to attack (this intention defined in model language as a constraint to commit forces) and then examined its optimal strategy to commit its remaining forces and found that such a strategy was instead a defense of the homeland, one can question red's feeling of superiority while committing an aggressive act. If red was acting rationally, he would be deterred if the outcome of the game (i.e., the determination of who would be the winner if red initiated the campaign) was obscure. If red was irrational, then blue would have to present a force posture to red much in excess of deterrence such that red's degree of irrationality could be effectively lowered to a rational level. Although deterrence as defined above is a weaker criteria than achieving a winning campaign objective, it is conceivable that one cannot be achieved without the other. In fact the analysis of the next section indicates that deterrence and overwhelming superiority for certain modes of fire power application are the same

Using the above definitions of deterrence, and considering the standard case of "A War in Southeast Asia" scenario as a basis of improving blue's prospects, the author will attempt to configure a reasonable limited war system and the many problems associated with strategic basing.

3. Firepower (Normalized Attrition Constant)

a. Incremental Firepower to all Terminal Access Areas (TA No. 0, 1, 2, 3)

In order to effectively counter red's overwhelming numerical superiority, a reasonable normalized attrition constant for blue must be determined. No other model parameter can effectively trade off this order of battle disparity. In order to keep the analysis at reasonable length, the number of troops assigned to the defender, blue, was made a constant equal to $.4 \times 10^6$, and all time delay inputs were kept the same as the standard case. To get the gross effect of the normalized attrition constant (which is proportional to relative firepower) on the scenario objectives (payoffs), a computer run varying a: $.001 \geq a \geq 1.0$ was made. The application of this incremental firepower by blue against red was made to all terminal access areas (TA No. 0, 1, 2, and 3). The results are presented in the same manner as the standard case above. Again, the unrestricted play of the game (first column, Figure 14) indicates both sides playing optimally with blue attacking and red defending TA No. 0. The firepower break-even point for blue is $a = .025$ as is expected from the model's use of Lanchester's Square Law for the differential law of combat. The campaign time t^* when $a = .025$, goes to infinity only mathematically; such a point in the solution of Lanchester's Equations being a singular point. In the second column of Figure 14, red is constrained to commit at least 10% of his forces to the invasion of TA No. 1, 2, and 3, and the second chart of this column indicates the nature of red's optimal decision concerning the rest of red's uncommitted forces. This chart has been isolated and is presented as Figure 15. For value of a: $0.001 \geq a \geq .014$ red wins the campaign no matter what decisions are made by either side concerning the optimal or non-optimal allocation of forces either to TA No. 0 and/or TA No. 1, 2, and 3. The optimal strategies available to red, once forced to commit at least 10% but not greater than 50% of his forces is obscure because they exist in mixed form. A solution of these mixed strategies would indicate red (and blue) would randomize their allocations to either the homeland (TA No. 0) or to the invasion areas (TA No. 1, 2, and 3) according to some fixed probability distribution. Both sides would

THE VARIATION OF BLUE'S FIREPOWER AS APPLIED TO TA #0,1,2,3

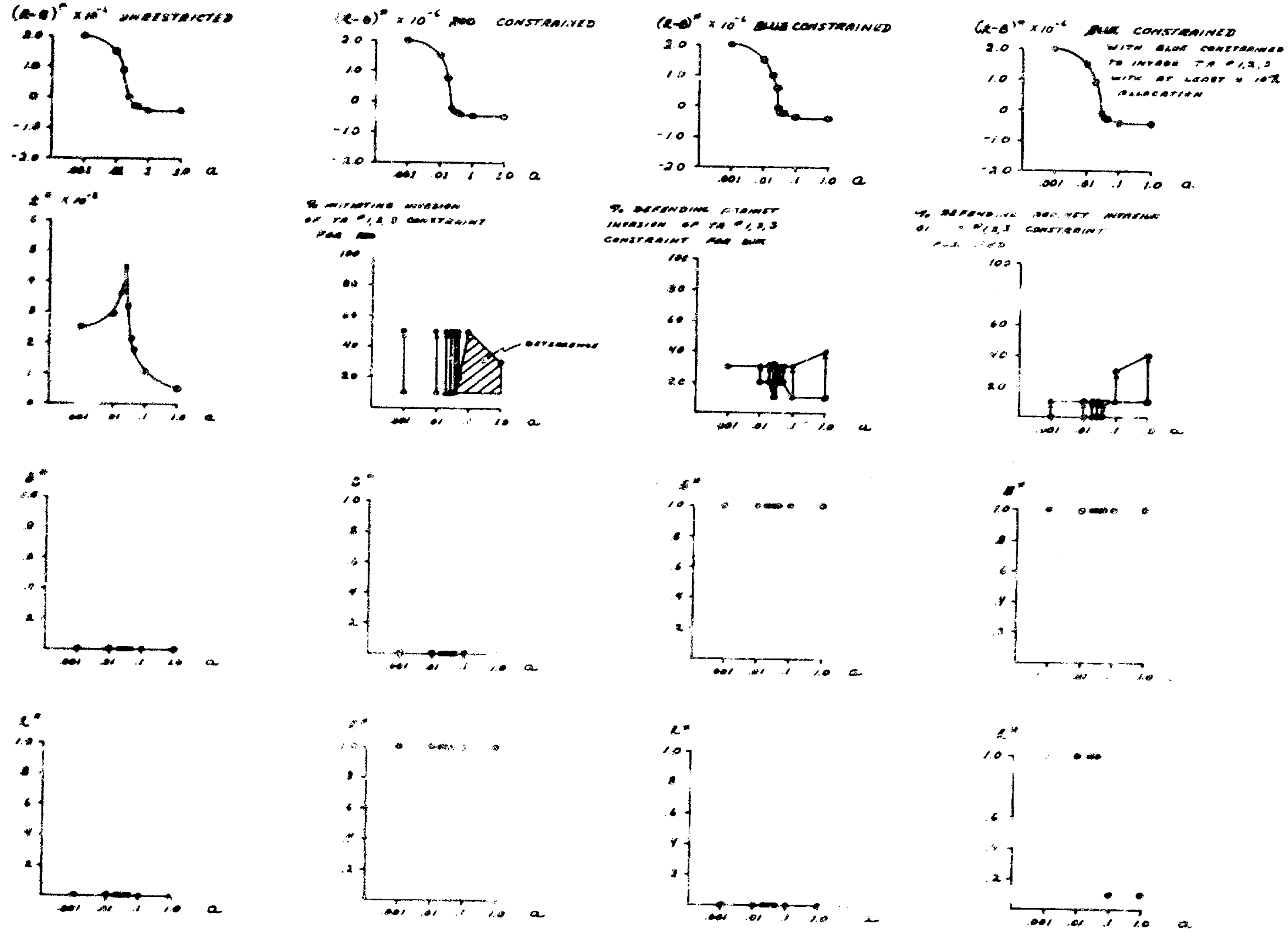


Figure 14

DECISION PROFILE FOR BOTH SIDES AS A FUNCTION OF BLUE'S NORMALIZED ATTRITION CONSTANT. (RED'S NORMALIZED ATTRITION CONSTANT = .001)

% INITIATING INVASION OF TA # 1, 2, 3 CONSTRAINT FOR RED

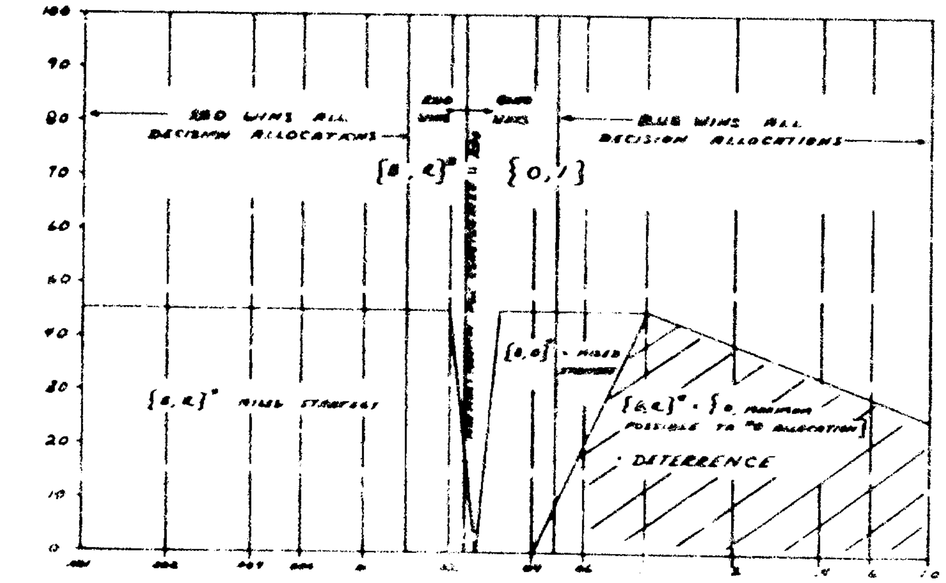
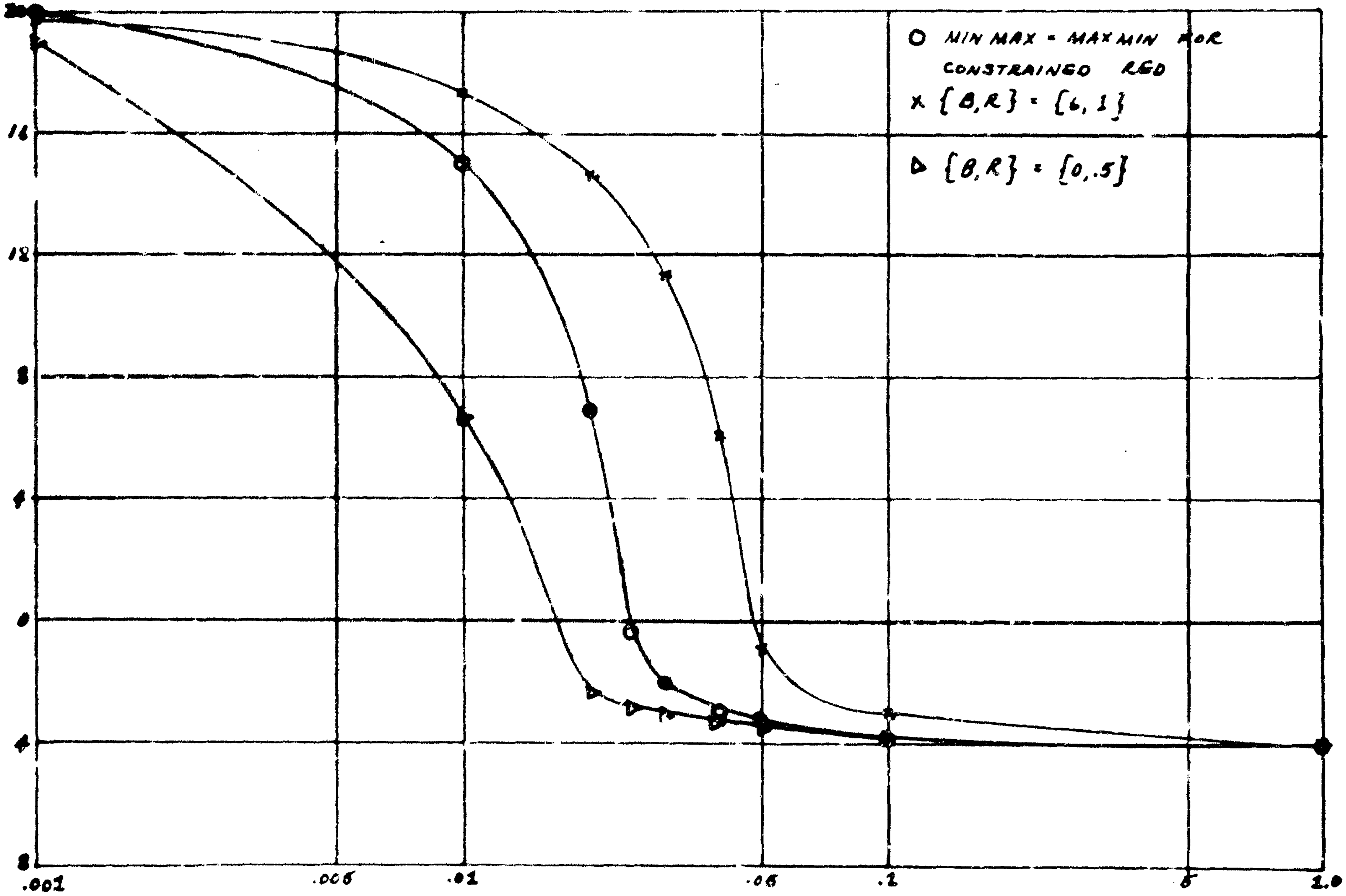


Figure 15

necessarily keep their random choice of allocation a secret to prevent the other side from taking advantage of the situation. One payoff would then be computed based on a population of expected moves which represent repeated plays of the game. Once red commits at least 50% of his forces to the terminal access areas (TA No. 1, 2, and 3), the strategies again become pure; (i.e., $\{B^*, R^*\} = \{0, 1\}$). However, in either case, if red is constrained or committed to attack with from 10% to over 50%, there is at least a probability that red will chose to commit all his forces to TA No. 1, 2 and 3. Obviously, deterrence as defined above, has not been accomplished in this range of values of a: $.001 \leq a \leq .014$ when red is forced to commit to TA No. 1, 2 and 3. The range of values, a: $.014 \leq a \leq .047$, shown in Figure 15 indicates the marginal play of the game where both must play optimally in order to guarantee (in the game-theoretic sense) a given payoff. To the left of the center line ($a = .0225$) of this region, red, the attacker, wins if the play is optimal, and to the right of this center line, blue, the defender, wins if the play is optimal. Non-optimal play by either side can cause the other side to gain a non-optimal advantage and result in a win. The decision surface in this region of a: $.014 \leq a \leq .047$ lies both above and below the datum plane. That is $(R - B)$ can be positive or negative depending upon the particular allocation each side uses. The three vertical lines defining this region are derived from values of $(R - B)$ taken from critical allocation decisions for both sides which determine the character of the decision surface (see Figure 16).

The optimal strategy for red, when forced to commit at least 10% of his forces in the domain of a: $.014 \leq a \leq .047$, is basically the same as the left-hand side of Figure 15, (i.e., mixed strategy to the 50% constraint level, and pure strategy beyond). Thus, we see that even if both sides were evenly matched, red, the attacker, would not be deterred from 'going all the way' once he had made a nominal (10% or better) commitment to TA No. 1, 2 and 3. In other words, for the particular scenario of "A War in Southeast Asia" being discussed, <u>blue does not deter red by presenting a force posture of parity against red.</u>

Figure 16 (R - B) FOR CRITICAL ALLOCATION DECISIONS FOR BOTH SIDES



Now, as we enter the range of value of a: $.047 \leq a \leq 1.0$, a new phenomena appears amongst the constrained optimal strategies available to red. Though red is forced to commit at least 10% of his troops to the terminal access areas (TA No. 1, 2, and 3), he no longer need project his remaining troops to the same area in order to achieve optimal strategy. Rather, his remaining forces may be committed to the defense of the homeland (TA No. 0). Thus it becomes apparent that to constrain red to invade the TA No. 1, 2 and 3 with at least 10% of his forces would not cause a rational red to commit 100%. Instead, red, to play optimally, must keep all the forces he can in the defense of the homeland (TA No. 0). This constrained decision also represents a pure strategy and is valid even if red is forced to commit from 30 - 50% of his forces to the TA No. 1, 2 and 3. In other words, blue has deterred red from reinforcing his allocation decision to 'go all the way'. The area of validity of this deterrence is illustrated by the shaded area of Figure 15. Notice this value of a: $.047 \leq a \leq 1.0$ represents an overwhelming superiority of blue over red as far as the outcome of the campaign is concerned as a function of blue's normalized attrition constant. Since red's normalized attrition constant has remained a = .001 throughout this analysis, blue requires: (1) a superiority factor of $(\frac{.0225}{.001}) = 22.5$ just to obtain parity (i.e., to make up for the disparity in order of battle) under optimal play; and (2) a factor of $(\frac{.1}{.001}) = 100$, or over four times the effective firepower of parity, for clearly defined deterrence. Needless to say, the range of values of 'a' pertinent to deterrence also assures blue's eventual victory no matter what strategies either side may use. In the real world, it is strongly implied that heavy firepower is required to guarantee deterrence. This requirement questions the value of using aircraft for strategic deployment of forces to combat areas if the airlift does not also contain the capability of deploying the necessary firepower along with such forces. Of course, the value of airlifting forces to overseas terminal access areas could be due to the increase in effectiveness of the operation derived from quick response time as measured by t_d and Δt in the model. These parameters will be analyzed in a later section.

The remaining analysis of the variation of the parameter firepower (as measured by 'a' and applied to TA No. , 1, 2 and 3) is shown in the third and fourth columns of Figure 14. It indicates a threshold of from 10% to 40% in stationing troops overseas in TA No. 1, 2, and 3. That is, when blue is constrained to station forces overseas in amounts of at least 30% of the total force level, then the optimal strategy is to station 100% of his forces in the TA No. 1, 2 and 3 (see second graph, third column, Figure 14). This constraint is true only when both blue and red have the same normalized attrition constant which means any aggressive action on red's part will result in red, the attacker, winning. This constraint drops to 20% as parity is reached. When deterrence is reached, blue's decision constraint is 10% for mixed strategies and changes to a pure strategy at 30%, that strategy being all forces allocated to TA No. 1, 2 and 3.

In the fourth column of Figure 14, red is forced to invade TA No. 1, 2 and 3 with at least 10% of his forces and blue's threshold for stationing all forces overseas is at least 10%. Again note that red has been deterred for values of $a \geq$ .. even though constrained to commit 10% of its forces (see last chart, fourth column, Figure 14).

b. Incremental Firepower to the Aggressor Terminal Access Area TA No. 0 Only

The last section assumed that superiority in directed firepower would be applied to all terminal access areas without constraint. If for real world reasons this increase in firepower were to be accomplished through the use of tactical nuclear weapons, then a political constraint might develop in which blue might want to apply such incremental force to the aggressor's homeland (TA No. 0) only. Our model applied such a constraint to a series of campaigns as defined by the war in the Southeast Asia Standard Case and the results are presented in Figure 17.

The results are suprisingly different from those obtained in the last section where unrestrained firepower was used. One major result of this run indicates that deterrence is achieved even before parity is

THE VARIATION OF BLUE'S FIREPOWER AS APPLIED TO TA #O

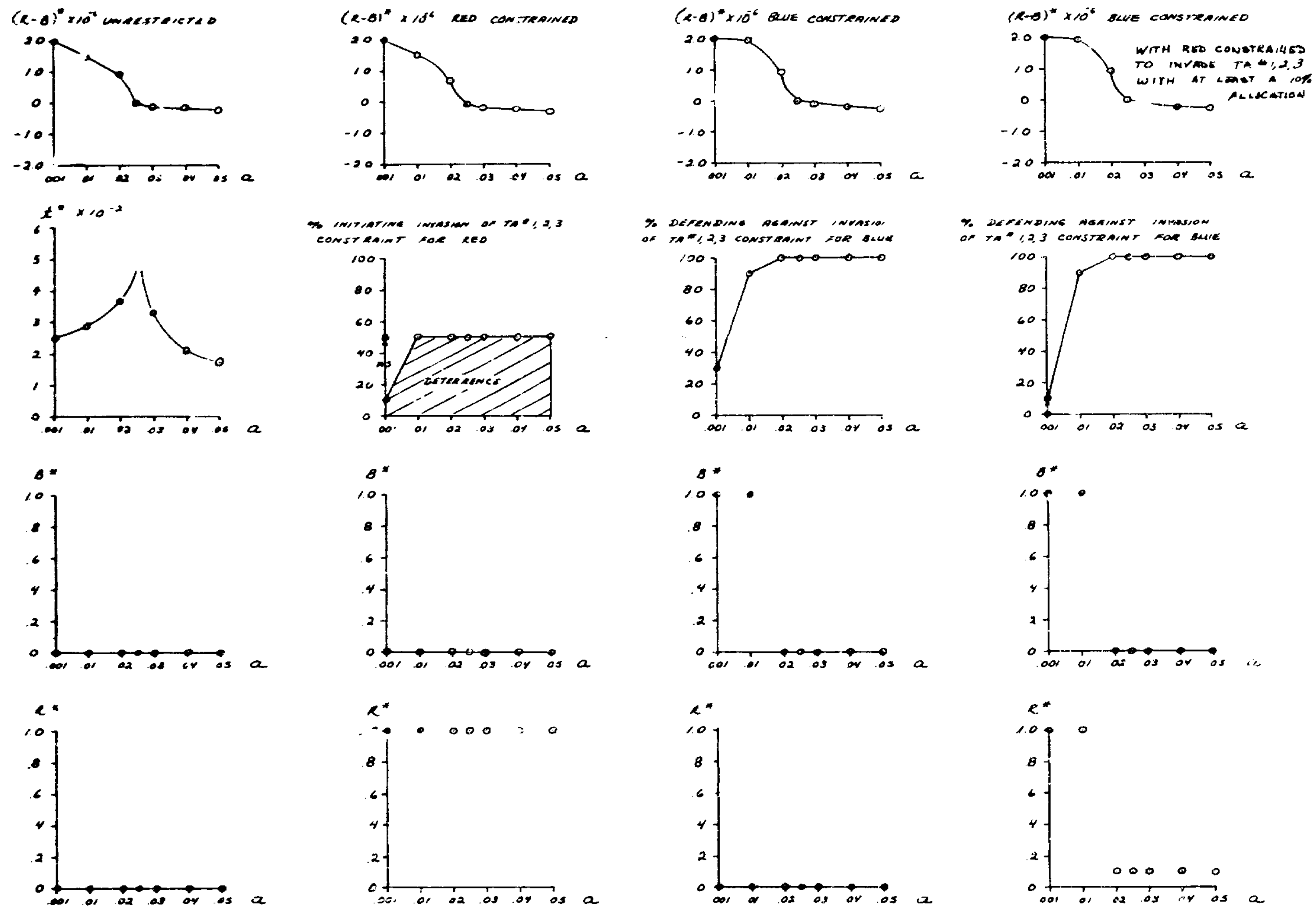


Figure 17

reached (see second chart, second column, Figure 17). Also, if blue achieved pairty or better, there would be no natural decision discontinuity in blue's stationing of forces overseas. That is, blue would always keep all the forces he could available for allocation to red's homeland. This conclusion assumes that blue will deploy such forces from the ZI (defined in the standard case). It is possible that deployment from an advance base is preferable, and this proposition will be analyzed later when parameters Δt and t_d are varied.

Notice, also, the lack of mixed strategies characteristic of the play in Figure 17 as compared with the results tabulated in Figure 14. Evidently, the localized application to TA No. 0 changes the shape of the decision surface such that strategies become crystal clear for both sides. Indecision due to marginal payoffs available to both sides (characteristic of mixed strategy) is removed. Secrecy concerning decision allocations is no longer required. This results in a considerable raising of blue's deterrence level even though a parity or superiority level between combatants will not exist unless blue attacks red's homeland in sufficient force. Thus, a policy limited to deterrence through increase in firepower must be compatible with blue'e intentions (policies) as demonstrated by a willingness to attack the homeland only.

c. The Effect of Firepower on the Strategic Basing of Limited War Forces

The effect of firepower, as represented by the variation of the normalized attrition constant, 'a', on the strategic basing of limited war forces has been indicated in Figures 14 and 17, with special emphasis on the third and fourth columns. These columns indicate blue's natural discontinuity decision levels when allocating forces either, (1) to the overseas TA No. 1, 2 and 3 before red initiates the invasion, or (2) to TA No. 0 projected from the zone of the interior (see Figures 11 and 12) after red initiates the invasion. The unrestricted play of the game indicates that both red and blue allocate forces such that all fighting is done at TA No. 0. This means blue will refrain from stationing forces overseas TA No. 1, 2 and 3 and deploy all her forces from the zone of the interior (ZI). The various time delays used in this Southeast Asia Scenario were based upon the projection of forces described above (see Figure 12). However, blue has a very low 'flip-flop' decision threshold (10%-30%: see second chart, third column, Figure 14) which directs him to station 100% of his forces overseas when constrained to maintain forces overseas equal to or greater than this threshold in order to defend against red's invasion of TA No. 1, 2 and 3. Notice that the threshold tends to go down as the normalized attrition constant available to blue goes up. Only where red is forced to invade with 10% of his forces and blue can muster enough firepower for deterrence, i.e., $a \geq .1$, do pure strategies exist which indicate no overseas basing necessary for blue (second and third graph, second column Figure 14). Otherwise a low order commitment by red to invade TA No. 1, 2 and 3, and a 0 to 10% stationing of blue's forces overasea at TA No. 1, 2 and 3 will force blue to use mixed strategies for optimal play (see second chart, fourth column, Figure 14). The application of a uniform increment of firepower over TA No. 0, 1, 2 and 3 indicates that blue suffers from an indecisive or indeterminate overseas basing policy. This policy requires the allocation either of all or none of blue's forces overseas, depending upon a low level of blue's possible precommitment of forces. As this precommitment level rises (above the 30 - 40% level constraint), blue adopts a very rigid overseas basing policy, (i.e., precommitting all of his troops overseas in order to play optimally).

On the other hand, blue's application of an increment of firepower toward red's homeland results in a very simple, consistent, and deterministic overseas basing policy for any level of overseas basing constraint with which blue must cope. This basing policy states that all forces not needed at TA No. 1, 2 and 3 should be stationed in the zone of the interior and projected to TA No. 0 after red initiates the invasion.

4. The Effect of Decreasing Blue's Response Time to Red's Invasion of TA No. 1, 2 and 3

Another parameter of the model of considerable real world importance is blue's response time, Δt, to red's invasion of the terminal access areas 1, 2 and 3. There are two methods where by blue can vary response time. One method of lowering Δt would be for blue to allocate more forces to advance bases before the invasion such that his forces would traverse a smaller distance to the TA No. 1, 2 and 3. The other method would be to utilize faster transport systems such as hydrofoil ships, aircraft, etc. Both these methods for affecting a lower response time are possible with the assault model. Figure 18 illustrates the Southeast Asia Scenario with variations of Δt and t_d which take into account the alternate use of surface, hydrofoil, jet aircraft, and rocket transportation. The force posture chosen for blue against red's invasion is one of deterrence and superiority. Uniform firepower superiority ($a = .1$) is applied by blue over all the terminal access areas, TA No. 0, 1, 2 and 3 and a 5 to 1 order of battle ratio in blue's favor is used. The results of this parameter variation were plotted as a function of B_0's deployment time in days (see Figure 19) utilizing the same format as have previous sections. The basic strategy for the unrestricted play of the game is as before, that is, each side allocates all their respective forces to TA No. 0 for response times Δt: $0 \leq \Delta t \leq 45$ days. However, if red is constrained to commit at least 10% of his forces to the terminal access areas 1, 2 and 3, then interesting things happen to red's natural discontinuity decision level as a function of response time. The second chart, second column of Figure 19, indicates that the percentage of red's forces initiating the invasion of TA No. 1, 2 and 3, is constrained by, and varies directly with, blue's response time to that invasion. From the standard case for readiness, embarkation/movement, and assault time of $\Delta t = 45$ days to a hypothetical response time of $\Delta t = 0$, red was less and less constrained to commit all his forces once he was constrained to commit some (from 50% down to 30%). From the real world point of view red was more likely to be deterred when blue presented a defensive force posture with a slower response time (all other parameters remaining constant). The abstract model on the other hand, interprets this situation in terms of the stability of the decision surface which in effect states that as blue's response time is lowered, red becomes more trigger happy and is less likely to maintain split forces if for some reason he was constrained to split them at the outset (forced to invade

UNITS	ORDER OF BATTLE TROOPS	READINESS EMBARKATION ASSAULT MOVEMENT Δt				RECOMMITMENT t_d				NORMALIZED ATTRITION	PRIORITY 1st	PRIORITY 2nd
R_0	$0 \rightarrow 2 \times 10^6$ in 10% intervals	0				0				.001	B_0	~
R_1	$\frac{2 \times 10^6 - R_0}{3}$ each	0				60				.001	B_1	B_0
R_2		0				60				.001	B_2	B_0
R_3		0				90				.001	B_3	B_0
		Surface Ships	Hydro foil	Jet Aircraft	Rocket	Surface Ships	Hydro foil	Jet Airc.	Rock.			
B_0	$0 \rightarrow .4 \times 10^6$	45	21	16	0	0	0	0	0	.1	R_0	~
B_1	$\frac{.4 \times 10^6 - B_0}{3}$ each	0	0	0	0	75	51	46	30	.1	R_1	R_0
B_2		0	0	0	0	75	51	46	30	.1	R_2	R_0
B_3		28	18	15	0	75	51	46	30	.1	R_3	R_0

Figure 18 SOUTHEAST ASIA SCENARIO - VARIATION OF BLUES RESPONSE AND RECOMMITMENT TIME WITH BLUE'S UNIFORM DISTRIBUTION OF INCREASED FIREPOWER TO TA NO. 0, 1, 2 and 3.

VARIATION OF BLUE'S RESPONSE TIME TO RED'S INVASION

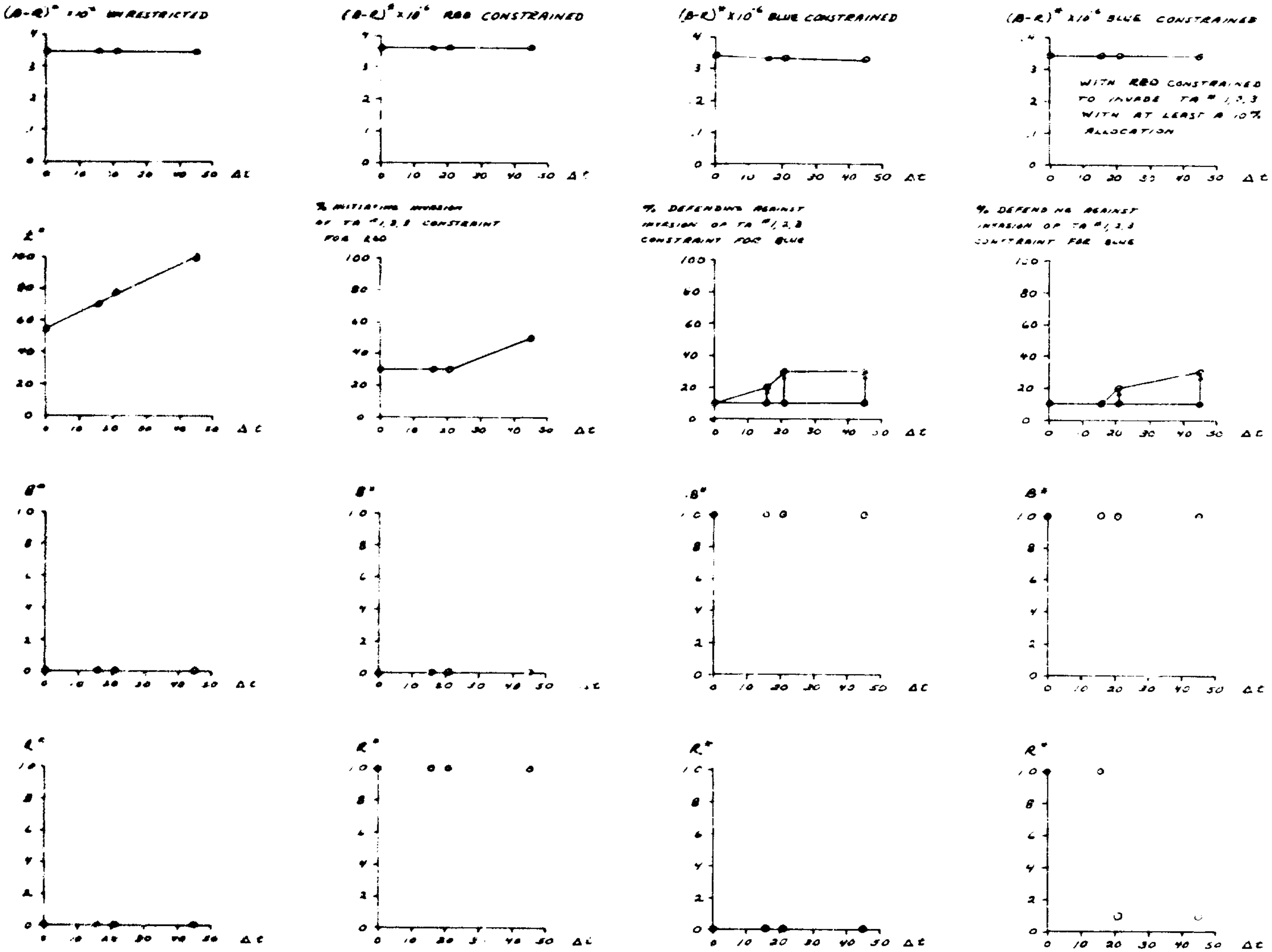


Figure 19

TA No. 1, 2 and 3). If these results are valid, one could question the entire concept of the airborne transfer of large numbers of limited war forces with or without their armament in response to an overseas threat. It should be noted that the above analysis considers the military payoff only. If a cost-effectiveness criteria were used, justification of the above airborne concept would be even harder to achieve.

As another result of the above analysis, one would also question the advisability of attempting to achieve a smaller reaction time in the defense of TA No. 3 (India) by creating an Indian Ocean Task Force. Certainly if this were done, the size of the task force would be prohibitive if the requisite number of troops and firepower were permanently stationed aboard.

Reviewing the third and fourth columns of Figure 19, we again see that an obscure low level, constrained; partial overseas basing policy on the defender's part leads to mixed strategies and then full commitment (100% deployment). The last chart of the last column of Figure 19 also indicates red's change of strategy as blue's response time is reduced when both blue and red are constrained to act.

Although they are not shown here, the results of the above analysis also apply if blue allocates his deterrent firepower to TA No. 0 only (Figure 20 indicates the scenario investigated).

UNITS	ORDER OF BATTLE TROOPS	READI-NESS / EMBARKATION MOVEMENT / ASSAULT Δt				RECOMMITMENT t_d				NORMALIZED ATTRITION	PRIORITY 1st	PRIORITY 2nd
R_0	$0 \rightarrow 2 \times 10^6$ in 10% intervals	0				0				.001	B_0	~
R_1	$\frac{2 \times 1^{6} - R_0}{3}$ each	0				60				.001	B_1	B_0
R_2		0				60				.001	B_2	B_0
R_3		0				90				.001	B_3	B_0
		Surface Ships	Hydro foil	Jet Aircraft	Rocket	Surface Ships	Hydro foil	Jet Airc.	Rocket			
B_0	$0 \rightarrow .4 \times 10^6$	45	21	16	0	0	0	0	0	.1	R_0	~
B_1	$\frac{.4 \times 10^6 - B_0}{3}$ each	0	0	0	0	75	51	46	30	.001	R_1	R_0
B_2		0	0	0	0	75	51	46	30	.001	R_2	R_0
B_3		28	10	15	0	75	51	[illegible]	30	.001	R_3	R_0

Figure 20 SOUTHEAST ASIA SCENARIO - VARIATION OF BLUE'S RESPONSE AND RECOMMITMENT TIME WITH BLUE'S APPLICATION INCREASED FIREPOWER TO TA NO. 0 ONLY

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IV. APPENDIX A

A BRIEF DESCRIPTION OF THE ASSAULT MODEL OF VOLUME I

A. PROBLEM DEFINITION, FORMULATION AND SYNOPSIS

Volume I demonstrated the use of analytical techniques to quantitatively describe the interrelationships among mobility, dispersion, surveillance, and fire power as they affect the survival of tactical units on the battlefield. The purpose of the study was to emphasize the possible use of analytical models to explore areas of Marine Corps/Navy advanced warfare military systems and operations, and ways in which outputs obtained from such analyses could lead, by implication, to recommendations for surveillance, fire power, force size, logistics, and command and control subsystems requirements.

The basic problem analyzed in this study can be summarized as follows:

An amphibious landing force, x, is to assault a limited area defended by a force y. The landing force is to be split up into an air mobile, x_a, and a surface mobile x_s. The defending force, in turn, allocates β_i: $i = 1$ to 2 of its force to each element of x where

$$\sum_{i=1}^{2} \beta_i = 1.$$

The questions to be asked are:

a. What allocation of forces should each side use against the other during the ensuing engagement?

b. What is the mathematical structure of the tactical decisions made by both sides (as defined by a) as a function of initial conditions (e.g., force levels (x, y) at time $t = 0$) and constraints (e.g., spatial and temporal limitations when allocating forces)?

c. How does the analysis relate back to the real world in terms of logistics, equipment, force levels, operational plans, etc.?

To abstract the analytic nature of combat during an amphibious operation from a scenario based upon the above descriptive analysis, one is tempted at first to start simply with a "Lanchester Equation" model approach. This model is by far the oldest analytic approach to land warfare and has great flexibility in its generalized form.

The form of the Lanchester Model which seems applicable is:

$$\frac{dx}{dt} = -by$$

$$\frac{dy}{dt} = -ax$$

$$x(0) = x_0$$

$$y(0) = y_0 \qquad \text{(A-1)}$$

where x_0 and y_0 represent the force levels of both sides at time $t = 0$ and a and b reflect each side's normalized attrition rate as seen by its opponent. Lanchester's Square Law* can be deduced from Equation (A-1) by taking the ratio of the two differential equations and integrating.

$$\frac{dx}{dy} = \frac{by}{ax} \qquad \text{(A-2)}$$

$$\int_{x_0}^{x} ax\, dx = \int_{y_0}^{y} by\, dy \qquad \text{(A-3)}$$

$$a(x_0^{\,2} - x^2) = b(y_0^{\,2} - y^2). \qquad \text{(A-4)}$$

Equation (A-4) indicates that the normalized attrition rate varies inversely as the square of the force level. This suggests the following transformation which will allow one to consider a force level and its attrition rate as an effective force level only.

*A survey of the literature referenced in Volume I indicates that reasonable agreement exists for Lanchester's Square Law as a model for the Pacific Island Campaign of World War II which was essentially an amphibious operation

$$x \rightarrow \frac{x}{\sqrt{a}}$$

$$y \rightarrow \frac{y}{\sqrt{b}}$$

$$t \rightarrow \frac{t}{\sqrt{ab}} \quad . \qquad \text{(A-5)}$$

This reduces Equation (A-1) to:

$$\frac{dx}{dt} = -y$$

$$\frac{dy}{dt} = -x$$

$$x(0) = x_0$$

$$y(0) = y_0 \quad . \qquad \text{(A-6)}$$

We are now in a position to relate this model formulation directly to the scenario of the amphibious assault problem described above. However, before we describe the mathematical model actually programmed, it might be useful to give a brief mathematical synopsis of this model utilizing the above notation in order to indicate the over-all direction of the analysis.

Using the dot notation for time derivatives, the amphibious operation can be described mathematically by the following equations:

$$
\begin{aligned}
\dot{x}_a &= -\beta y \\
\dot{x}_s &= -(1-\beta) y \\
\left.\begin{aligned}
\beta \dot{y} &= -x_a \\
(1-\beta)\,\dot{y} &= -x_s
\end{aligned}\right\} &\quad x = x_a + x_s \\
\left.\begin{aligned}
&x(0) \\
&y(0)
\end{aligned}\right\} &\quad \text{given at } t = 0
\end{aligned}
\tag{A-7}
$$

where $\beta;\ 0 \le \beta \le 1$ is the allocation of that part of y's force to x_a and $(1 - \beta)$ to x_s, and $\{x_s, x_a\}$ represents an allocation of all of the amphibious landing force x to either the surface or air mobile mode of attack.

If any unit on either side is reduced to zero during the battle, the opposing side's remaining allocation against that unit will be assumed to be transferable for combat against the other side's remaining units. The winner will be the side that survives the battle. To relate force levels and normalized attrition rates, the following scaling laws apply:

$$
\begin{aligned}
x_a &\longrightarrow \sqrt{a_a}\; x_a \\
x_s &\longrightarrow \sqrt{a_s}\; x_s \\
y &\longrightarrow \sqrt{b}\; y \\
t &\longrightarrow \sqrt{a_i b}\; t; \quad i = a, s.
\end{aligned}
\tag{A-8}
$$

The model actually programmed can be extended to include x's reserve forces x_r which can be thrown into the action at some later time. This model also takes into consideration the differences in fire power available to x when projected through a vertical envelopment assault or via a surface beachhead landing. That is, x's Regimental Landing Teams,(RLT's) are projected ashore as quickly as possible with the heavy ordnance coming after. In the above model it is assumed that x has many units, each with its own characteristic fire power. The greater normalized attrition rate, a_s, as associated with the surface force can only be projected ashore at a given rate. The vertical envelopment with the lower attrition rate, a_a, can be assumed to be projected ashore instantaneously. In this way x is faced with an allocation choice of projecting into battle low fire power forces x_a at a high rate (instantaneously) or a high fire power force x_s at a much lower rate.

It is felt that these limiting assumptions are justified in that we are concerned only with the essence of the problem at this stage of the study; that is, with the gross outcome of the battle as a function of input parameters $\{x(0);\ y(0)\}$, decision parameters $\{x_a(0),\ \ x_s(0);\ \ \beta\}$, and constraints $\{a_s,\ a_a;\ b;$ time to initiate battle$\}$. Hopefully, this initial analysis will develop useful insight into the structure of amphibious operations such that more significant models can be developed later.

Summarizing, Equation(A-7)represents a simple model of an amphibious operation in a game theoretic context which attempts to relate the interplay of force levels, allocation decisions, and constraints as they affect the outcome of the battle. The payoff of such a game will be defined by the solutions of the above equations as a function of time at the specific time when one side's force level is either reduced to zero or, any finite level. The remaining side's force level represents the value of the game, positive to the winner and negative to the loser. This is called a zero-sum, two-person game.

Another possible zero-sum, two-person game based upon the above model could be contructed by defining the payoff function or value of the game as the time t^* it takes to complete the amphibious assault successfully; that is, the time it takes to reduce the opposing side either to zero or any predetermined level. The attacker would attempt to minimize this time while the defender would attempt to maximize it. Strategies for both sides, defined by this type of payoff function, could have a realistic interpretation in that one of the primary purposes of an amphibious operation is to secure a beachhead as quickly as possible so that the main body of attacking forces can be placed ashore safely. The defending forces hinder the landing force as long as possible, so that superior reserves deployed elsewhere can be brought to bear upon the attacker while he is in the vulnerable position of establishing a beachhead. The amphibious landing force will have superiority at a beachhead providing the time between the committing of the attacking forces and the securing of the beachhead is less than the time necessary for the defender to effectively deploy his reserves against that beachhead. The outcome of such a battle, using the above model, can then define the effectiveness of deception techniques employed by the attacker when selecting a beachhead and planning the invasion. The model could also determine the relationship and value of fire power, mobility, dispersion, reaction time, and surveillance to various proposed deception techniques.

B. THE AMPHIBIOUS OPERATION MODEL

When constructing an amphibious operation model, abstraction of the physical process of an amphibious assault must first be formulated. Certain items, such as number of battle units, force levels, firepower, etc., may be considered as inputs to the physical process while other items such as casualties, duration of the battle, etc., may be viewed as outputs. In mathematical terms, the amphibious operation is an "operator" with the physical input variables as its domain and the physical output variables as its range. In view of the complexity of a general amphibious operation and the extremely large number of contingent possibilities that can arise during the execution of any individual operation, it would be exceedingly difficult if not impossible to completely represent the operator mathematically. Our task then is to construct a mathematical operator that is an approximation of the physical process. The nature of the approximation is determined by the uses to which the model will be put and the resultant simplifications that can be tolerated without materially affecting the significant results.

As the purpose of this study is to qualitatively determine the nature of optimal battle strategies under very general conditions, no attempt will be made to abstract the analytic nature of the amphibious operation via an all-inclusive model. The basic unit of force for the protagonists, blue and red, will be taken as a battle unit and the battle as a whole will be viewed as an aggregation of local conflicts among individual battle units. Thus, at any one instant of time, we need only consider a series of relatively simple local conflicts to determine the state of the battle as a whole. Since we will assume that the dynamics of any local conflict are governed by Lanchester's Equations, the only pertinent information is the composition of the local conflicts and the force levels and attrition constants for the individual battle units. This information may be summarized in a list of the following form:

	Unit	Force Level	Attrition Rate
	R1	-	-
	R2	-	-
	.		
	.		
	.		
	Ri	-	-
Local Conflict No. 1			
	B1	-	-
	B2	-	-
	.		
	.		
	.		
	Bj	-	-
	Ri + 1	-	-
	Ri + 2	-	-
	.		
	.		
	.		
	Rk	-	-
Local Conflict No. 2			
	Bj + 1	-	-
	Bj + 2	-	-
	.		
	.		
	.		
	Bm	-	-
etc.			

The composition of the local conflicts will be determined by an Opponent Priority List (OPL) for each battle unit. In order to reduce the complexity of the computer program that implements this model, the decision was made to limit the OPL to two levels, first priority opponents and second priority opponents. The following behavior is then postulated for the individual battle units:

RULE 1: A battle unit will seek to engage his first priority opponents if they have not already been eliminated from the battle. If a unit is engaged with an opponent, that opponent is termed a direct opponent of the given unit.

RULE 2: If a unit's first priority opponents have been eliminated, he will, after a specified time delay, seek to engage his second priority opponents.

RULE 3: Any two units that share a common direct opponent are considered to be battle allies.

RULE 4: All the direct opponents of a given unit's battle allies are taken to be additional direct opponents of the unit itself.

These rules offer great flexibility to the conflicts since a unit can be drawn into a given conflict in many ways - by attacking an opponent who is involved in that conflict, by being attacked by an opponent who is involved in that conflict, or by sharing a common direct opponent with an ally who is involved in that conflict.

In general, the application of Rules 1 and 2 and the repeated application of Rules 3 and 4 will completely determine the composition of all the local conflicts. (A formal proof can be constructed which shows that if Rules 3 and 4 are applied a limited number of times, a unique splitting up of the units into local conflicts results. The proof, which will be omitted here, rests on the fact that the above rules define an equivalence relation on the set of all units and this relation completely partitions the set into equivalence classes.)

Within each local conflict, the progress of the battle may be measured by means of Lanchester's Equations. These differential equations can be solved analytically and evaluated to determine the force levels of any unit in a given local conflict at any time in the future, as long as the basic composition of that local conflict remains unchanged. Once the basic composition changes, the coefficients of the differential equations change, and the original analytic solution is no longer valid. At this point we must redetermine the composition of the local conflicts, calculate the new coefficients for the differential equations, and begin a new set of analytic solutions to continue where the old solutions left off.

The composition of a local conflict can be changed one of two ways:

1. a unit is added to the battle
2. a unit is eliminated.

The specification of a time of arrival for each unit determines the time at which the unit is added to the battle. A single time of arrival is given since it is postulated that once a force is committed to the battle it will not be withdrawn and recommitted at another time. Once a unit is added, it, of course, seeks out its opponents according to Rules 1 and 2.

The elimination of a unit occurs when its force level drops below some specified minimum force level. The time at which this takes place can be determined by inverting the analytic solutions of the differential equations and solving for time as a function of force levels. This inverse solution has several possible forms depending on the coefficients and initial values of the differential equations.

After all the first priority opponents of a given unit are eliminated, Rule 2 specifies a time delay before that unit may engage its second priority opponents. (The rule also applies if the first priority opponents have been eliminated prior to the unit's time of arrival.) This delay factor is intended to reflect the geographical location of the units and their relative mobilities.

In summary, the following items are the necessary input parameters for each unit:

1. initial force level
2. minimum force level
3. attrition constants
4. time of arrival
5. time delay factor
6. first priority opponents list
7. second priority opponents list.

The application of Rules 1, 2, 3 and 4 in conjunction with the preceding items, determines the composition of the local conflicts while Lanchester's Equations give the force level of each of the units as a function of time and indicate the time at which a unit is eliminated.

As analytic methods are used throughout and no time-step simulation is utilized, this method furnishes us with an extremely rapid means of determining the expected outcome (or "payoff") of the battle, determined by the given battle plans and initial force levels.

If various elements of the battle plans and/or initial force levels are considered as parameters, we are in a position to generate trade-off tables showing the effect of a variation in one or more of the parameters; for example, the degradation of payoff due to increased time spacing of the various landing groups. Furthermore, game theoretic techniques may be used to simultaneously optimize the choice of parameters for both the attacker and defender.

C. PROBLEM ILLUSTRATION

To illustrate the methodological techniques for handling an amphibious landing situation, a specific problem will be investigated (see Figure a). Blue, B, an amphibious landing force, is to assault a limited area defined by red R. The landing force is to be split up into three surface mobile elements, B_1, B_2, B_3 and one air mobile element, B_4, each of which is to initiate action sequentially. At short intervals, δt, three waves of surface elements are projected ashore in the following order: (1) infantry battalion type units, B_1; (2) infantry and close support artillery type units, B_2; and (3) infantry and a tank section, B_3. The air mobile element consisting of a vertical envelopment team of infantry and close support artillery units, B_4, is projected inland, a distance d from the beach (see Figure a) at the same time B_1 arrives across the beach. The relative firepower of these four elements of blue are assumed to be in the following ratio: $B_1 : B_2 : B_3 : B_4 :$ $1.0 : 2.0 : 4.0 : 1.5$. The red defending force, R, in turn, commits its forces either to the beach, R_1, to the air mobile attack, R_2, or to both, at the same time B_1 and B_4 are deployed. The relative firepower of these two elements of red are assumed to be equal to blue's maximum, i.e., $B_3 : R_1 : R_2 : 1.0 : 1.0 : 1.0$. Note (from Figure a) that each element of blue and red has a predetermined battle commitment time which reflects the amphibious landing force's logistic constraints. This time of commitment is symbolized by the clock next to each element in Figure a. Also, take note that the ensuing battle takes place at two different locations; the beach and some inland point. The model developed for this problem reflects this spatial characteristic of the battle by defining a set of time delays $\left\{t_{d_i}\right\}$ which applies to each force element of the battle. These time delays indicate the amount of time which would be required for an element to traverse the distance from one battle area to the other during the course of the battle. An element traverses this distance only if it has a first priority opponent which it destroys, and any second priority opponents can only be reached by traversing the distance d shown in Figure a. If a force element does not have an opponent at its initial point on the battlefield,

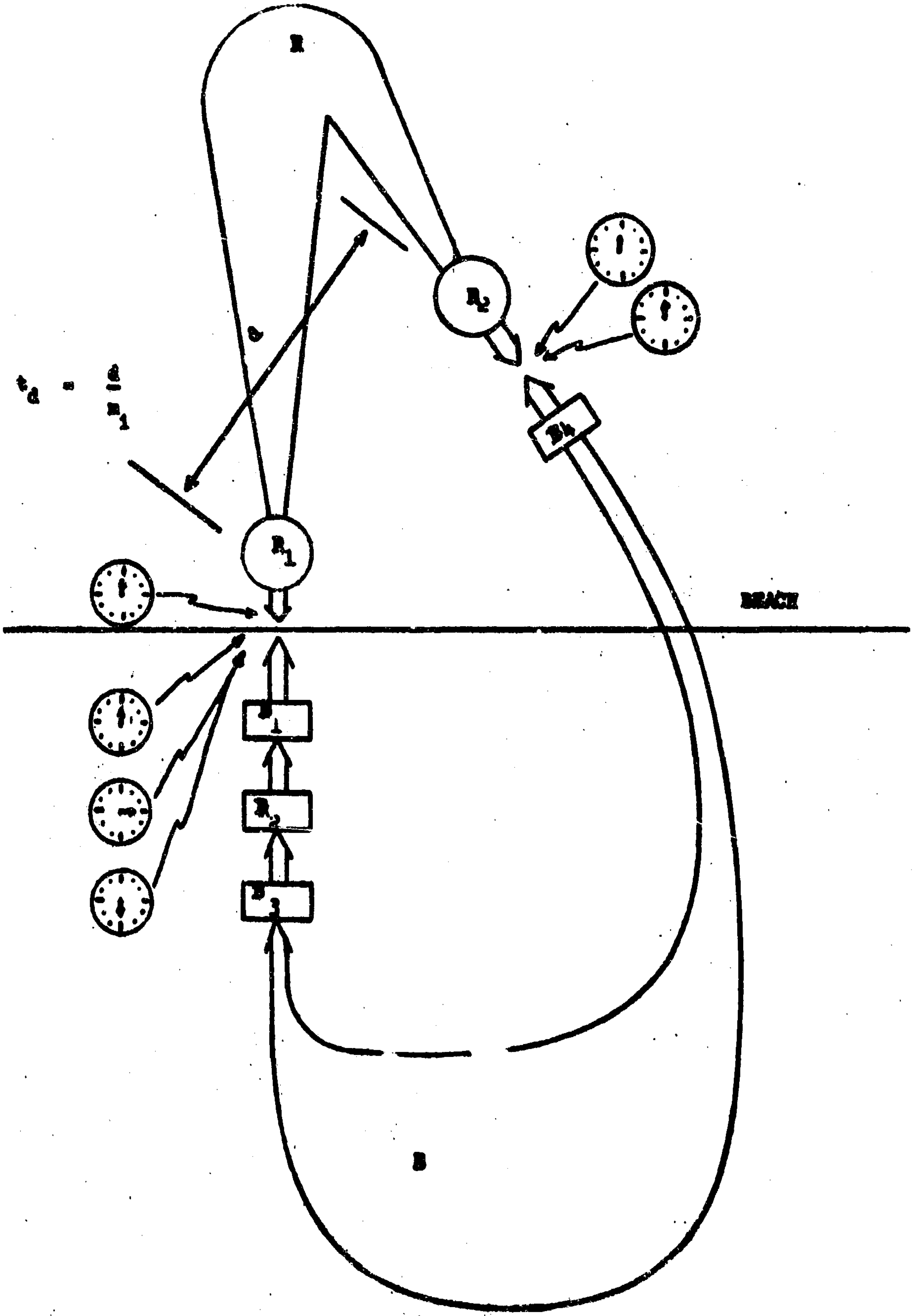

Figure a THE AMPHIBIOUS LANDING MODEL

the time delay t_d will denote the time necessary to meet an opponent located at the other point on the battlefield. Naturally, this time delay reflects the mobility characteristics of the units involved and represents an important trade-off parameter.

Another aspect of the problem not indicated in Figure a is that each fighting element has an Opponent Priority List (OPL), which indicates the order in which each opponent is to be attacked. In some cases, fighting elements are placed in position to protect the flanks of an operation. These elements never initiate an attack and are drawn into a conflict only when the opposing side's priority initiates battle action against the flank.

The following questions are pertinent to the above problem description:

a) What is the mathematical structure of the tactical decisions made by both sides in selecting the action variables such as number of forces, time delays, battletime commitments, priorities, etc.?

b) How does this type of an analysis relate back to the real world?

c) What allocation of forces should each side use in initiating the battle? For example both red and blue must allocate their units to the beach and/or inland locations. Both sides must make such allocations quantitatively.

d) What time sequence should the amphibious landing force adhere to in projecting its elements into combat?

The significant input variables to the model based upon the above scenario are:

a) order of battle ratios - the total number of troops each side has available for the operation

b) firepower - each fighting element's rate of kill per man per hour

c) time sequencing of units into the battle

d) battle priorities

e) unit mobility factors.

The significant output variables are:

a) the number of survivors at the end of the battle

b) the duration of the battle for any threshold of defeat of one side

c) the optimum allocation strategies available to each side. (That is, what fraction of the total forces goes across the beach or deployed air mobile as opposed to the defender's allocation of what fraction of his forces in defending the beach as opposed to the forces kept in reserves to defend against the air mobile attack?)

d) The natural strategic discontinuity levels, i.e., the areas of the strategy surface beyond which both sides must play to obtain strategic optimality.

e) Sub-optimum strategies (restricted within discontinuity levels), i.e., areas of the strategy surface, if both sides were constrained within, strategic optimality cannot be achieved and sub-optimal strategies become important.

The last two outputs reflect the constraining nature of the real world situation. Mathematically optimal strategies may not always be achieved because of the physical constraints of the tactical systems employed; e.g., landing craft and helicopter capacity, speed of operation, and duty cycle of the logistic support systems, etc. However, it should be noted that the model does not delineate these constraints directly, but rather defines the mathematical structure of the decisions made in terms of natural discontinuity levels on the decision surface. These discontinuities in turn define the areas in which optimal strategies would shift violently if the real world were for any reason constrained to operate in these areas only. The strategies derived from this shift are sub-optimum.

We are now in a position to discuss some of the details of the solution, e.g., optimality, sub-optimality, decision surface, mathematical structure, etc. The results of the battle, as defined above and in Figure b, are displayed as a mathematical surface; i.e., a single valued function of two variables. The ordinate of the surface is called the payoff of the game played between blue and red and represents the number of blue survivors at the time red is destroyed. This is a function of the fraction of total forces allocated by blue to air mobile attack and by red in defense against air mobile attack (see Figure b). Thus, a payoff matrix is generated which mathematically represents a two-dimensional surface for each possible decision of either side (see Figure b). The height of this surface above the grid

PAYOFF MATRIX

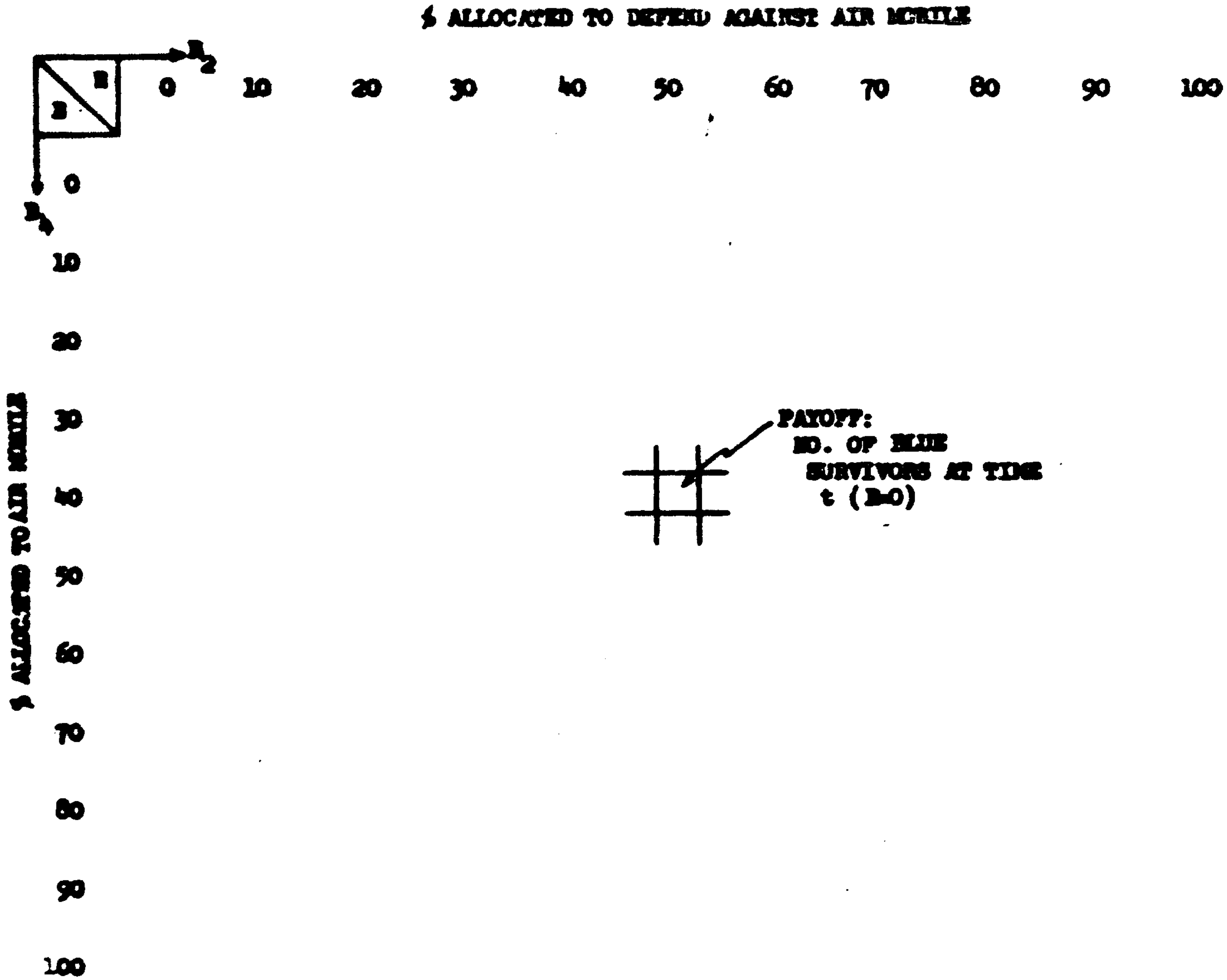

Figure b Tactical Decision Surface

of decision variables (i.e., the fraction allocated to air mobile attack and to defense against air mobile attack) indicates the number of survivors left on the victorious side. This height is defined as positive if blue wins and negative if red wins. Since blue's payoff is the opposite of red's payoff, and it is logical to expect blue to try to maximize the payoff function while red does its best to minimize this same function, we have a two-person zero-sum game with the allocation fractions for both sides representing strategic variables. The criteria that will be used to determine the optimality of the strategies will be the min-max payoff for red and max-min payoff for blue. When min-max equals max-min, a pure solution exists, and the payoff corresponding to this pure solution is called the value of the game. If no such equality exists, then it can be shown that min-max >max-min, and the only way to get equality is to redefine the payoff function as an expected value of survivors with the players picking their strategic variables (i.e., fraction of total forces allocated to air mobile and defending against air mobile) according to some probability distribution. Decisions made in this manner are called mixed strategies and usually represent marginal strategies for the side having to employ them. From the tactical system design or requirements point of view, one would never knowingly initiate an amphibious operation against a defending force which depended upon a mixed strategy to gain the objective of the operation. This would be tantamount to having to bluff in order to achieve success. The logical plan for an amphibious operation would be to land with overwhelming superiority and allow any advantage accrued by the maintenance of secrecy in initiating the operation to compensate for faulty threat intelligence estimates, acts of God (e.g., bad weather that is unpredictable), etc.

To obtain the game-theoretic solution to the amphibious operation described above (see Figure a) we first compute the mathematical surface representing the payoffs of all possible allocations for both sides (Figure b). Then we designate the minimum of each row with an ellipse and the maximum of each column with a rectangle (Figure c). Blue will select the maximum of the minimum tagged in the rows while

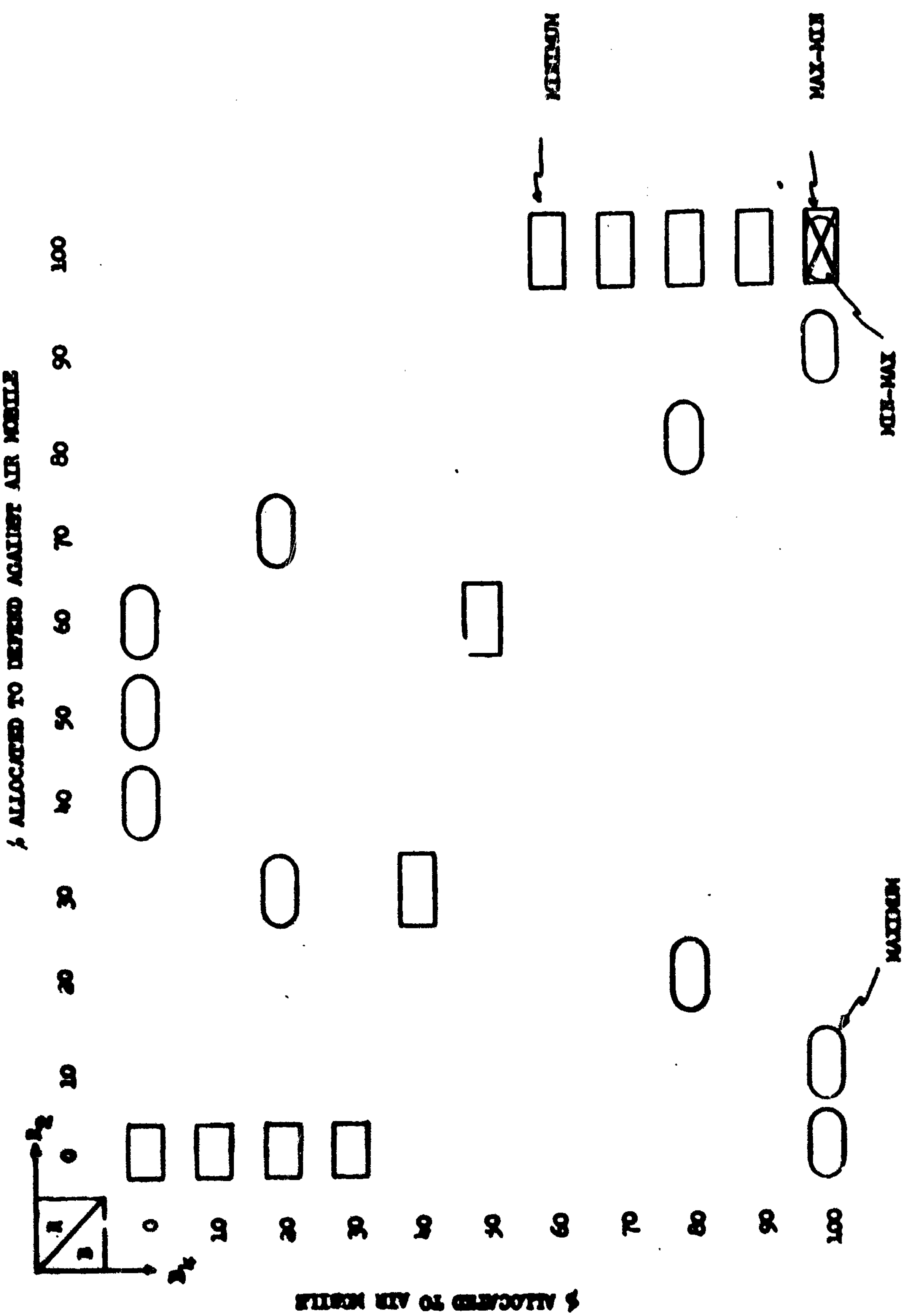

Figure c Game Theoretic Solution - Unrestricted Case red will select the minimum of the maximum tagged in the columns. In Figure c, when the maximum of the minimum payoff and the minimum of the maximum payoff occur at the same point, then the value of the game is defined as the number of survivors of blue/red (positive for blue, negative for red) located at this point on the surface. The strategies associated with this point are pure. In most cases the min-max = max-min solution occurs at corners of the matrix.

Once the unconstrained value of the game and associated decision variables are known, the solutions determined from natural discontinuities of the surface are generated. This is accomplished by placing a constraint raster first on blue's strategy line starting from the 100% allocation decision level and moving the raster toward the 0% allocation. For each placement of the raster the matrix is solved for the min-max = max-min solution in the same manner as outlined above, but only for the partitioned matrix (see Figure d) consisting of blue's strategy line from 0% allocation to the raster and red's unrestricted strategy line. This sub-optimum solution to the game is then related to the placement of the raster by noting when the solution changes abruptly as the raster moves from 100% to 0% allocation. For example, if blue's unrestricted strategy is to allocate 100% of the fighting elements to the air mobile or vertical envelopment decision, the raster is then placed under the 90% level restricting blue's strategy from 0% to 90% and the solution to this restricted game is noted. If the optimum solution is the 90% allocation of the fighting elements via the air mobile mode decision, then this is considered to be no change in the basic strategy. That is, the restricted game still demands that blue send all his fighting units air mobile even though blue is restricted by the 90% allocation level. After this determination, the raster is then moved to the 80% allocation level and the restricted game again solved. If the solution yields the 80% allocation level decision the raster is moved to the 70% level, etc. In most cases during this process of methodically constraining blue's decision level, the game theoretic solution will abruptly change to yield an optimum decision other than the maximum possible fighting units going air mobile. The

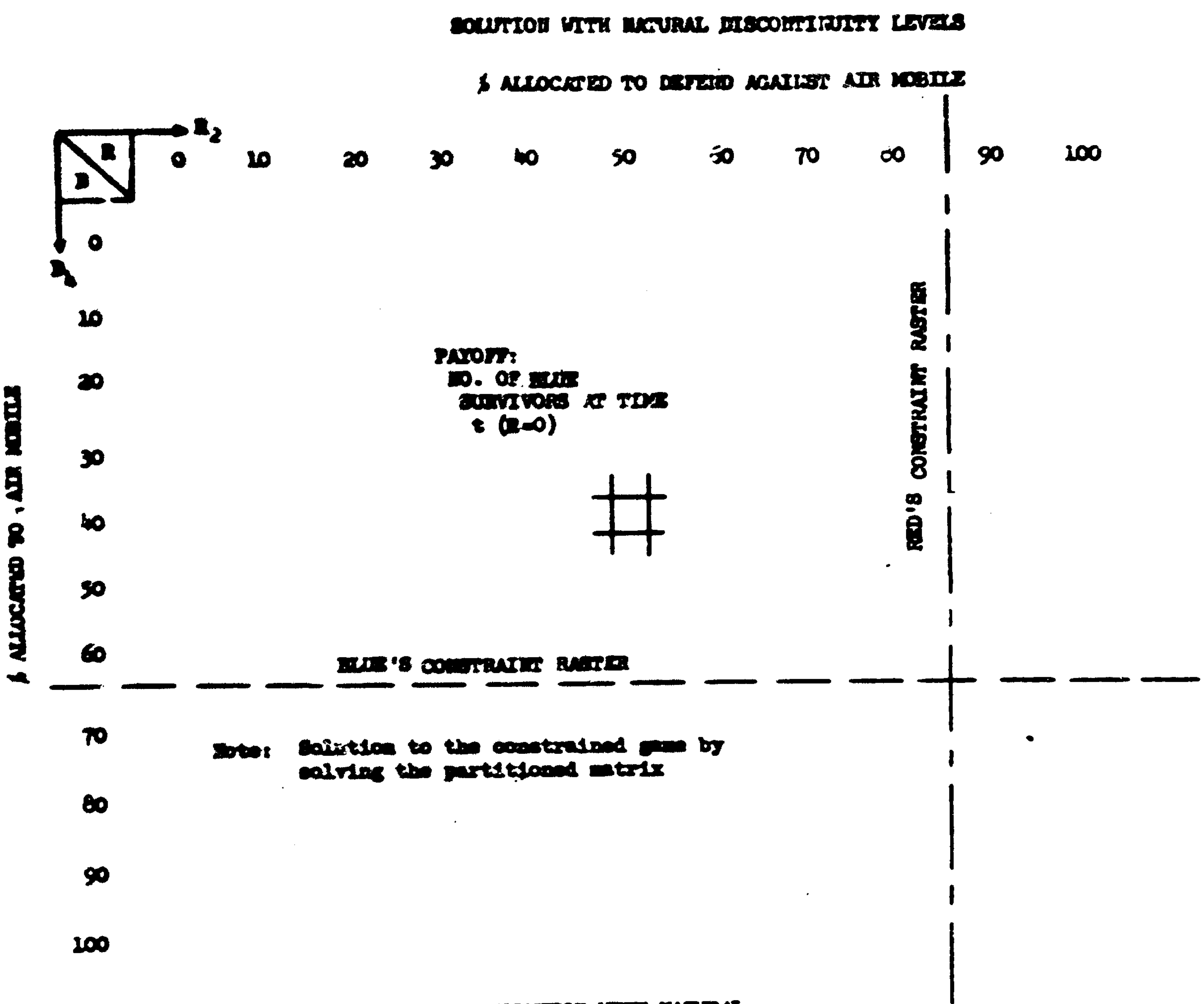

Figure d GAME THEORETIC SOLUTION WITH NATURAL DISCONTINUITY LEVELS position of blue's constraint raster when such an abrupt decision occurs is then recorded and the associated sub-optimum strategy restricted within this partitioned matrix is called a natural strategic discontinuity level. The physical meaning of such sub-optimum strategies is that if for any physical reason blue cannot send 100% of his forces air mobile (the optimal policy), at what level of decision constraint must blue change his tactics completely concerning a given mode of attack. Therefore, the natural strategic discontinuity level represents a threshold in blue's strategic thinking, above which blue will attack in the vertical envelopment mode with all the fighting elements he physically can get air mobile, and below which he will use the sub-optimum strategy based upon the solution of the partitioned matrix. An example of this threshold from the real world would be the decision of a commander not to send an air mobile strike in support of an across-the-beach operation if he felt that the air mobile forces would not be able to act as a fighting unit during the time necessary for the main body of forces from the beach to join with the air mobile group for projecting the battle inland. There are many historical incidences of commanders in World War II and in the Korean War making this type of decision.

It is significant that the mathematical model used for this problem contains such a threshold without the analyst being aware, a priori, of its presence. This tends to confirm the validity of the model as a realistic abstraction of the amphibious operation. It will also be interesting to see the quantification of these thresholds as a function of the various inputs to the model tabulated above would yield significant information that could be used in the design and analysis of present, presently planned and future tactical systems.

Getting back to the formal development of the Problem Illustration, everything said about blue's constraint raster vis-a-vis red is equally true for red's constraint raster vis-a-vis blue (see Figure d). And finally, once both blue's and red's natural strategic discontinuity levels are known, a solution to the game is obtained by solving the partitioned payoff matrix with each player operating within his natural constraints at the same time. Figure d shows this decision surface as the area in the upper left hand corner of the matrix.

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13. ABSTRACT

Objectives of Study: to demonstrate the use of analytical techniques to quantitatively describe the inter-relationships between mobility, dispersion, surveillance and firepower, as they affect the survival of tactical units on the battlefield. The purpose of such a study was to emphasize the possible use of analytical models to explore areas of Marine Corps/Navy advanced warfare military systems and operations, in which outputs obtained from such analysis would lead, by implication, to recommendations for requirements for surveillance, firepower, force size, logistics, and command and control subsystems.

Scope of Study: Two phases of limited war amphibious operations are analyzed for the long range period:

(a) The assault

(b) The strategic basing of limited war forces.

The assault phase includes the deployment of troops, during an amphibious operation, across the beach and/or via the vertical envelopment mode against the opposition of the defense. The strategic basing phase includes the deployment of tactical forces throughout the world and the requirements placed upon such deployment as a function of the campaign objectives pertinent to the many potential theatre of operations.

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